



Maths Calculation Policy

Person responsible: Headteacher
Ratified by governing body: Summer 2021
Date for review: Spring 2024

A handwritten signature in black ink, which appears to read 'Anne-Marie Renshaw'. The signature is written in a cursive style with a long, sweeping underline.

Reverend Anne-Marie Renshaw
Chair of Governors

Maths calculation Policy

This policy supports the Hamilton Trust maths scheme used throughout the school.

Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum.

This calculation policy should be used to support children to develop a deep understanding of number and calculation. This policy has been designed to teach children through the use of concrete, pictorial and abstract representations.

By having a clear and consistent curriculum strategy throughout the school, teachers can ensure that children are hearing consistent language and using progressive methods that build from one year to the next. This policy shows the written calculation methods together with suggested Concrete, Pictorial & Abstract methods (CPA).

Concrete — a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.

Pictorial – a pupil can now relate the concrete to representations, such as a diagram or picture of the problem.

Abstract — a pupil is now capable of representing problems by using mathematical notation, for example $12 \times 2 = 24$.

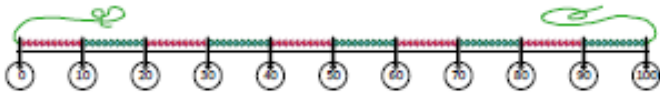
Written Calculation methods Years 1-6

Addition and subtraction are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using + and two using -) which can be written to express the relationship between 4 and 6 and 10. It is key to a good understanding of addition and subtraction that $6 + [] = 10$ and $10 - 6 = []$ are seen as ways of expressing the same question.

+ Addition

Using place value

Count on in ones/counting in tens, e.g. knowing $45 + 1$ or $45 + 10$ without counting on in ones.



$45 + \square = 50$ $65 + \square = 70$

$85 + \square = 90$

Counting on

Count on in ones, e.g. $11 + 2 =$ and $7 + 4 =$
Count on in tens, e.g. $45 + 20$ as 45, 55, 65

Using number facts:

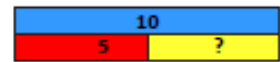
'Story' of 4, 5, 6, 7, 8 and 9, e.g. $7 = 7 + 0$ or $6 + 1$ or $5 + 2$ or $4 + 3$.
Number bonds to 10, e.g. $5 + 5$, $6 + 4$, $7 + 3$, $8 + 2$, $9 + 1$, $10 + 0$.



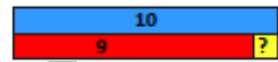
$6 + \square = 10$



$7 + \square = 10$



$5 + \square = 10$



$9 + \square = 10$

Patterns using known facts, e.g. $4 + 3 = 7$ so we know $24 + 3$, $44 + 3$, $74 + 3$, etc.

Bead strings and 1-100 number grid help counting on/back in tens.

- Subtraction

Using place value

Count back in 1s/Count back in 10s.
Say one less than any number to 100.
Say 10 less without counting back in ones.

1	2	3	4	5
11	12	13	14	15
21	22	23	24	25
31	32	33	34	35
41	42	43	44	45

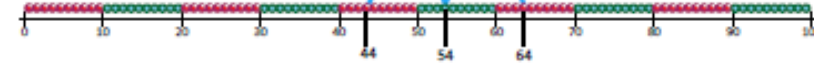
$33 - 10 = 23$

Subtracting by taking away

Count back in ones,
e.g. $15 - 3 =$ $25 - 3 =$



Count back in tens.

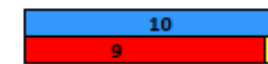


Using number facts:

'Story' of 4, 5, 6, 7, 8 and 9, e.g. $7 - 1 = 6$, $7 - 2 = 5$, $7 - 3 = 4$, etc.
Number bonds to 10, e.g. $10 - 1 = 9$, $10 - 2 = 8$, $10 - 3 = 7$, etc.



$10 - \square = 7$



$10 - \square = 9$

Missing number sentences, $3 + \square = 7$, link addition and subtraction.

Patterns using known facts,
e.g. $10 - 7 = 3$ so we know $30 - 7 = ?$

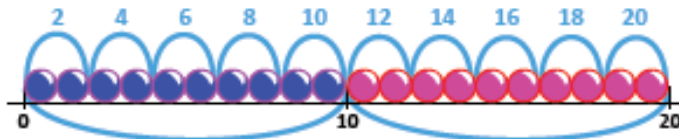


Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $\square \times 5 = 45$ and $45 \div 5 = \square$ are seen as ways of expressing the same question.

x Multiplication

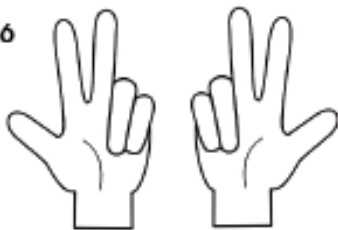
Counting in steps ('Clever' counting)

Count in 2s and 10s.



Doubling and halving

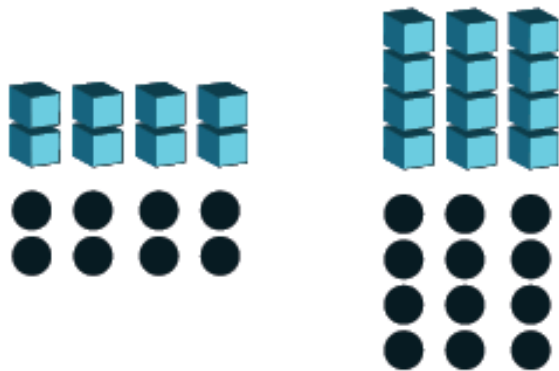
Find doubles to double 6 using fingers.



'Clever' counting is an excellent basis for multiplication and division.

Grouping

Begin to use visual and concrete arrays and 'sets of' objects to find the answers to '3 lots of 4' or '2 lots of 5', etc.

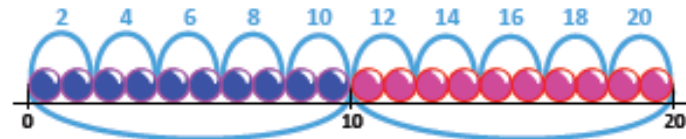


Division must be presented as the inverse of multiplication (grouping).

÷ Division

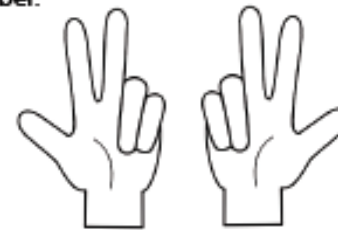
Counting in steps ('Clever' counting)

Count in 2s, and 10s.



Doubling and halving

Find half of even numbers up to 12 including realising that it is hard to halve an odd number.



Grouping

Begin to use visual and concrete arrays and 'sets of' objects to find the answers to 'how many towers of 3 can I make with 12 cubes?'

Sharing

Begin to find half of a quantity using sharing, e.g. half of 16 cubes by giving one each repeatedly to two children.



Addition and subtraction are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using + and two using -) which can be written to express the relationship between 4 and 6 and 10. It is key to a good understanding of addition and subtraction that $6 + [] = 10$ and $10 - 6 = []$ are seen as ways of expressing the same question.

+ Addition

Using place value

Know 1 more or 10 more than any number, e.g. 1 more than 67 or 10 more than 85.

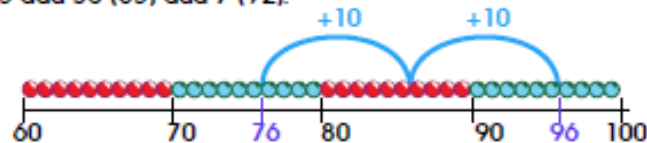
Partitioning, e.g. $55 + 37$ as $50 + 30$ and $5 + 7$ finally combining the two totals: $80 + 12$.

$$\begin{array}{r} 50 \\ + 30 \\ \hline 80 \end{array} + \begin{array}{r} 5 \\ + 7 \\ \hline 12 \end{array} = 80 + 12 = 92$$

Bead strings and 1-100 number grid help counting on/back in tens.

Counting on

Add ten and multiples of ten, e.g. $76 + 20$ as $76, 86, 96$ or in one hop $76 + 20$. Add two 2-digit numbers by counting on in tens and then in ones, e.g. $55 + 37$ as 55 add 30 (85) add 7 (92).



Add near multiples, e.g. $46 + 19$ or $63 + 21$.

Using number facts

Know pairs of numbers which make the numbers up to and including 10, e.g. $8 = 4 \& 4, 3 \& 5, 2 \& 6, 1 \& 7$ and $10 = 5 \& 5, 4 \& 6, 3 \& 7, 2 \& 8, 1 \& 9, 0 \& 10$. Patterns of known facts, e.g. $6 + 3 = 9$, so we know $36 + 3 = 39$, $66 + 3 = 69$, $53 + 6 = 59$.

Bridging ten, e.g. $57 + 5$ as 57 add 3 then add 2 more.



Adding three or more single-digit numbers, spotting bonds to 10 or doubles, e.g. $6 + 7 + 4 + 2$ as $10 + 7 + 2$.

Missing number sentences, $3 + [] = 7$, link addition and subtraction.

- Subtraction

Using place value

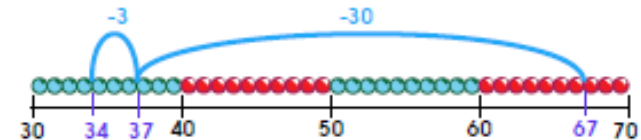
Know 1 less or 10 less than any number, e.g. 1 less than 74 or 10 less than 82.

Partitioning, e.g. $55 - 32$ as $50 - 30$ and $5 - 2$ combining the answers: $20 + 3$.

$$\begin{array}{r} 50 \\ - 30 \\ \hline 20 \end{array} + \begin{array}{r} 5 \\ - 2 \\ \hline 3 \end{array} = 20 + 3 = 23$$

Taking away

Subtract ten and multiples of ten, e.g. $76 - 20$ as $76, 66, 56$ or in one hop $76 - 20 = 56$. Subtract two 2-digit numbers by counting back in tens then in ones, e.g. $67 - 33$ as 67 subtract 30 (37) then count back 3 (34).

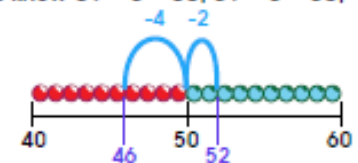


Subtracting near multiples, e.g. $74 - 21$ or $57 - 19$.

Using number facts

Know pairs of numbers which make the numbers up to and including 10, e.g. $10 - 6 = 4, 8 - 3 = 5, 5 - 2 = 3$, etc. Patterns of known facts, e.g. $9 - 6 = 3$, so we know $39 - 6 = 33, 69 - 6 = 63, 89 - 6 = 83$.

Bridge ten, e.g. $52 - 6$ as 52 subtract 2 then subtract 4 more.



Counting up

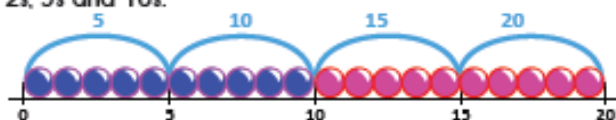
Find a difference between two numbers on a line, e.g. $51 - 47$.

Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $\square \times 5 = 45$ and $45 \div 5 = \square$ are seen as ways of expressing the same question.

\times Multiplication

Counting in steps ('Clever' counting)

Count in 2s, 5s and 10s.



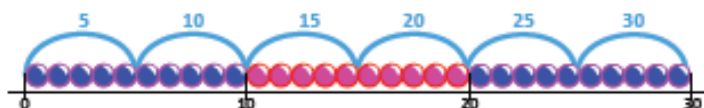
Begin to count in 3s.

Doubling and halving

Begin to know doubles of multiples of 5 to 100, e.g. *double 35 is 70.*

Grouping

Use arrays to find answers to multiplication and relate to 'clever' counting, e.g. *3 x 4 as three lots of four things and 6 x 5 as six steps in the 5s count as well as six lots of five.*



Understand that 5×3 can be worked out as three 5s or five 3s.

Use number facts

Know doubles to double 20

$$\text{Double } 7 = 14$$



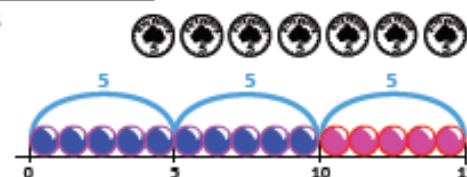
Division, grouping, is the inverse of multiplication.

Start learning 2x, 5x, 10x tables, relating these to 'Clever counting' in 2s, 5s, and 10s, e.g. *5 x 10 = 50, and 10, 20, 30, 40, 50 is five steps in the tens count.*

\div Division

Counting in steps ('Clever' counting)

Count in 2s, 5s and 10s



Doubling and halving

Find half of numbers up to 40, including realising that half of an odd number gives a remainder of 1 or an answer containing a $\frac{1}{2}$. Begin to know half of multiples of 10 to 100, e.g. *half of 70 is 35.*

Grouping

Relate division to multiplication by using arrays of towers of cubes to find answers to division, e.g. *how many towers of five cubes can I make from 20 cubes as $\square \times 5 = 20$ and also as $20 \div 5 = ?$*



Relate division to 'clever' counting and hence to multiplication, e.g. *how many 5s do I count to get to 20?*

Sharing

Begin to find half or a quarter of a quantity using sharing, e.g. $\frac{1}{4}$ of 16 cubes by sorting the cubes into four piles. Find $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of small quantities.

half of 20 is...

20	
?	?

Using number facts

Know halves of even numbers to 24. Know 2x, 5x and 10x division facts. Begin to know 3x division facts.

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+ Addition

Using place value

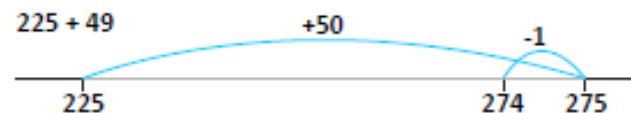
Count in hundreds, e.g. knowing $475 + 200$ as 475, 575, 675.

Add multiples of 10, 100 and £1,
e.g. $746 + 200$ or $746 + 40$ or
 $£6.34 + £5$ as $£6 + £5$ and 34p.

Partitioning, e.g. $68 + 74$ as $60 + 70$ and $8 + 4$ and
combine the totals: $130 + 12 = 142$
Or $£8.50 + £3.70$ as $£8 + £3$ and $50p + 70p$ and combine: $£11 + £1.20$.

Counting on

Add 2-digit numbers by adding the multiple of ten then the ones,
e.g. $67 + 55$ as 67 add 50 (117) add 5 (122).
Add near multiples of 10 and 100, e.g. $67 + 39$ or $364 + 199$.



Count on from 3-digit nos, e.g. $247 + 34$ as $247 + 30$ (277)
then $277 + 4 = 281$.

Using number facts

Number bonds to 100, e.g. $35 + 65$, $46 + 54$, $73 + 27$, etc.

100	
65	35

Add to next ten and next hundred, e.g. $176 + 4 = 180$, $435 + 65 = 500$, etc.

- Subtraction

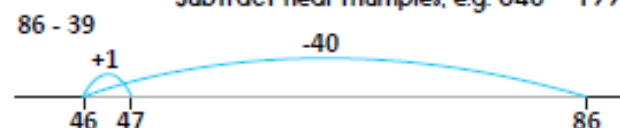
Taking away

Use place value to subtract, e.g. $358 - 300$ or $348 - 40$ or $348 - 8$.
Taking away multiples of 10, 100 and £1, e.g. $476 - 40 = 436$,
 $476 - 300 = 176$, $£4.76 - £2 = £2.76$.

Partitioning, e.g. $68 - 42$ as $60 - 40$ and $8 - 2$ or
 $£6.84 - £2.40$ as $£6 - £2$ and $80p - 40p$.

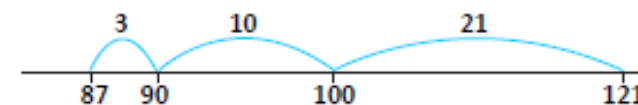
Count back in hundreds, tens and then ones,
e.g. $763 - 121$ as $763 - 100$ (663) then subtract 20 (643)
then subtract 1 (642).

Subtract near multiples, e.g. $648 - 199$ or $86 - 39$.



Counting up

Find a difference between two numbers by counting up from the smaller
to the larger, e.g. $121 - 87$.



Using number facts

Number bonds to 100, e.g. $100 - 35 = 65$, $100 - 48 = 52$, etc.

100	
48	?

Subtraction is both taking away and - importantly - difference.

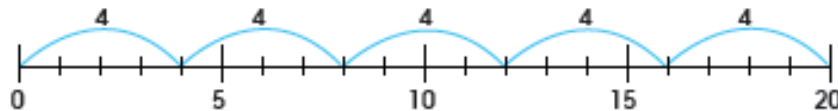
We no longer count in 1s but use PV and number facts.

Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $[\] \times 5 = 45$ and $45 \div 5 = [\]$ are seen as ways of expressing the same question.

\times Multiplication

Counting in steps ('Clever' counting)

Count in 2s, 3s, 4s, 5s, 8s and 10s, e.g. colour the multiples on a 1-100 grid or use hops along a landmarked line.



Doubling and halving

Find doubles to double 50 using partitioning. Use doubling as a strategy in multiplying by 2, e.g. 18×2 is double 18 (36).

$$\begin{array}{r} 48 \\ 80 + 16 = 96 \end{array}$$

Grouping

Recognise that multiplication is commutative, e.g. $4 \times 8 = 8 \times 4$. Multiply multiples of 10 by single-digit numbers, e.g. $30 \times 8 = 240$. Multiply friendly 2-digit numbers by single-digit numbers, e.g. 13×4 .

Using number facts

Know doubles to 20 and doubles of multiples of 5 to 100, e.g. double 45 is 90. Know doubles of multiples of 5 to 100, e.g. double 85 is 170. Know 2x, 3x, 4x, 5x, 8x, 10x tables facts.

\times Written Multiplication

Build on partitioning to develop grid multiplication.

\times	20	3	=
4	80	12	92

Doubling and halving form the basis of mental \times & \div strategies.

Number facts must be memorised and used on a daily basis.

\div Division

Counting in steps ('Clever' counting)

Count in 2s, 3s, 4s, 5s, 9s and 10s by colouring numbers on the 1-100 grid or using a landmarked line.



Doubling and halving

Find half of even numbers to 100 using partitioning. Use halving as a strategy in dividing by 2, e.g. $36 \div 2$ is half of 36.

$$\begin{array}{r} 36 \\ 15 + 3 = 18 \end{array}$$

Grouping

Recognise that division is not commutative, e.g. $16 \div 8$ does not equal $8 \div 16$. Relate division to multiplications 'with holes in', e.g. $[\] \times 5 = 30$ is the same calculation as $30 \div 5 = ?$ thus we can count in in 5s to find the answer. Divide multiples of 10 by single-digit numbers, e.g. $240 \div 8 = 30$.

Using number facts

Know halves of even numbers to 40.

28	
?	?

Know halves of multiples of 10 to 200, e.g. half of 170 is 85.

Know 2x, 3x, 4x, 5x, 8x, 10x division facts.

Use division facts to find unit and simple non-unit fractions of amounts within the times tables, e.g. $\frac{3}{4}$ of 48 is $3 \times (48 \div 4)$.

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+ Addition

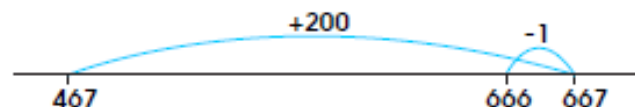
Using place value

Count in thousands, e.g. knowing $475 + 200$ as 475, 575, 675.
Partitioning, e.g. $746 + 203$ as $700 + 200$ and $46 + 3$
or $134 + 707$ as $130 + 700$ and $4 + 7$.

PV and number facts are central to mental strategies.

Counting on

Add 2-digit numbers by adding the multiple of ten then the ones, e.g. $67 + 55$ as 67 add 50 (117) add 5 (122).
Add near multiples of 10, 100 and 1000, e.g. $467 + 199$ or $3462 + 2999$.



Count on to add 3-digit numbers and money, e.g. $463 + 124$ as $463 + 100$ (563) $+ 20$ (583) $+ 4 = 587$ or $£4.67 + £5.30$ as $£9.67$ add 30p.

Using number facts

Number bonds to 100 and to next multiple of 100, e.g. $463 + 37$, $1353 + 47$.

Number bonds to £1 and to the next whole pound, e.g. $£3.45 + 55p$.
Add to the next whole number, e.g. $4.6 + 0.4$ or $7.2 + 0.8$.

Counting up is essential for money calculations and, later, decimals.

- Subtraction

Taking away

Use place value to subtract, e.g. $4748 - 4000$ or $4748 - 8$, etc.

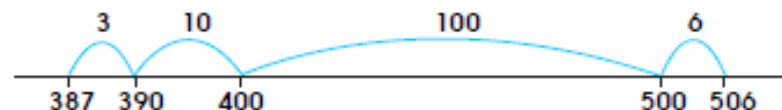
Take away multiples of 10, 100, 1000, £1, 10p or 0.1, e.g. $8392 - 50$ or $6723 - 3000$ or $£3.74 - 30p$ or $5.6 - 0.2$.

Partitioning, e.g. $£5.87 - £3.04$ as $£5 - £3$ and $7p - 4p$ or $7493 - 2020$ as $7000 - 2000$ and $90 - 20$.

Count back, e.g. $6482 - 1301$ as $6482 - 1000$, then $- 300$, then $- 1$ (5181).
Subtract near multiples, e.g. $3522 - 1999$ or $£34.86 - £19.99$.

Counting up

Find a difference between two numbers by counting up from the smaller to the larger, e.g. $506 - 387$.



$$100 + 10 + 6 + 3 = 119$$

Using number facts

Number bonds to 10, 100 and derived facts, e.g. $100 - 76 = 24$, $1.0 - 0.6 = 0.4$.

100	
76	24

Number bonds to £1 and £10, e.g. $£1.00 - 86p = 14p$ or $£10 - £3.40 = £6.60$.

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+ Written Addition

Build on expanded column addition to develop compact column addition with larger numbers.

$$\begin{array}{r} 1000 \ 400 \ 60 \ 8 \\ + 4000 \ 800 \ 60 \ 6 \\ \hline 1000 \ 100 \ 10 \\ \hline 6000 \ 300 \ 30 \ 4 \end{array}$$

Compact column addition with larger numbers.

$$\begin{array}{r} 5347 \\ 2286 \\ + 1495 \\ \hline 121 \\ \hline 9128 \end{array}$$

Use expanded and compact column addition to add amounts of money, e.g. $\pounds 3.24 + \pounds 2.58$.

$$\begin{array}{r} \pounds 3 \ 20\text{p} \ 4\text{p} \\ + \pounds 2 \ 50\text{p} \ 8\text{p} \\ \hline \pounds 5 \ 70\text{p} \ 12\text{p} \ \pounds 5.82 \end{array} \qquad \begin{array}{r} \pounds 3.24 \\ + \pounds 2.58 \\ \hline \pounds 5.82 \end{array}$$

Add like fractions, e.g. $\frac{3}{8} + \frac{1}{8} + \frac{1}{8}$.

Expanded methods firm up a robust understanding of place value.

Stress that decimals and fractions are parts of a whole.

- Written Subtraction

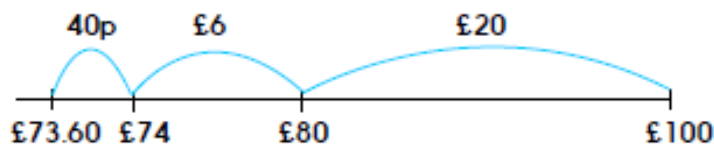
Expanded column subtraction.

$$\begin{array}{r} 600 \ 110 \ 16 \\ ~~700~~ \ ~~20~~ \ ~~8~~ \\ - 300 \ 50 \ 8 \\ \hline 300 \ 60 \ 8 \end{array}$$

Begin to use column subtraction.

$$\begin{array}{r} 6 \ 11 \ 16 \\ - 3 \ 5 \ 8 \\ \hline 3 \ 6 \ 8 \end{array}$$

Use counting up subtraction to find change from $\pounds 10$, $\pounds 20$, $\pounds 50$ and $\pounds 100$, e.g. $\pounds 100 - \pounds 73.60$.



$$\pounds 20 + \pounds 6 + 40\text{p} = \pounds 26.40$$

Subtract like fractions, e.g. $\frac{3}{8} - \frac{1}{8} = \frac{2}{8}$.

Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $[\] \times 5 = 45$ and $45 \div 5 = [\]$ are seen as ways of expressing the same question.

\times Multiplication

Counting in steps (sequences)

Count in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 11s, 12s, 25s, 50s, 100s and 1000s.

Doubling and halving

Find doubles to double 100 and beyond using partitioning, e.g. double 226.

$$\begin{array}{c} 226 \\ \swarrow \quad \searrow \\ 400 + 40 + 12 = 452 \end{array}$$

Begin to double amounts of money,

e.g. £3.50 doubled is £7.

Use doubling as a strategy in multiplying by 2, 4 and 8,

e.g. $34 \times 4 =$ double 34 (68) doubled again (136).

Grouping

Use partitioning to multiply 2-digit numbers by single-digit numbers.

Multiply multiples of 100 by single-digit numbers using tables facts,

e.g. $400 \times 8 = 3200$.

Multiply using near multiples by rounding, e.g. 24×19 as $(24 \times 20) - 24$.

Using number facts

Know times tables up to 12×12 .

Facility in doubling and halving is key for mental \times and \div strategies.

Stress that division is multiplication with 'holes' in.

\div Division

Counting in steps (sequences)

Count in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 11s, 12s, 25s, 50s, 100s and 1000s.

Doubling and halving

Find halves of even numbers to 200 and beyond using partitioning.

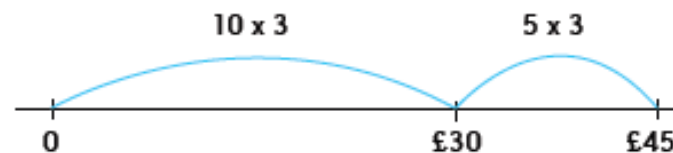
344	
172	172

Begin to half amounts of money, e.g. £9 halved is £4.50.

Use halving as a strategy in dividing by 2, 4 and 8, e.g. $164 \div 4$ is half of 164 (82) halved again (41).

Grouping

Use multiples of 10 times the divisor to divide by numbers < 9 above the tables facts, e.g. $45 \div 3$.



Divide multiples of 100 by single-digit numbers using division facts, e.g. $3200 \div 8 = 4000$.

Using number facts

Know times tables up to 12×12 and all related division facts.

Use division facts to find unit and non-unit fractions of amounts within the times tables, e.g. $\frac{7}{8}$ of 56 is $7 \times (56 \div 8)$.

Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $\square \times 5 = 45$ and $45 \div 5 = \square$ are seen as ways of expressing the same question.

\times Written Multiplication

Use grid multiplication to multiply 3-digit by 1-digit numbers.

$$\begin{array}{c|c|c|c} \times & 200 & 50 & 3 \\ \hline 6 & 1200 & 300 & 18 \end{array} = 1518$$

If children understand place value they can develop fluency.

Use a vertical written algorithm (ladder) to multiply 3-digit numbers by 1-digit numbers.

$$\begin{array}{r} 253 \\ \times 6 \\ \hline 1200 \\ 300 \\ 18 \\ \hline 1518 \end{array}$$

\div Written Division

Written version of a mental method:

$$\square \times 3 = 86$$

$$86 \div 3 = \underline{28} \text{ r } 2$$

$$\begin{array}{r} 20 \times 3 = 60 \\ \hline 26 \end{array}$$

$$\begin{array}{r} 8 \times 3 = 24 \\ \hline 2 \end{array}$$

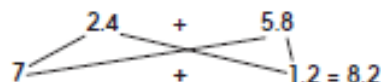
Addition and subtraction are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using + and two using -) which can be written to express the relationship between 4 and 6 and 10. It is key to a good understanding of addition and subtraction that $6 + [] = 10$ and $10 - 6 = []$ are seen as ways of expressing the same question.

+ Addition

Using place value

Count in 0.1s, 0.01s, e.g. knowing what 0.1 more than 0.51 is.

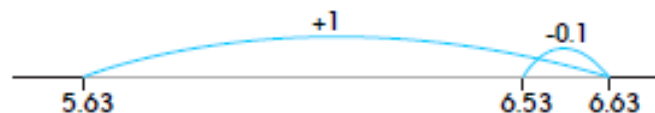
Partitioning, e.g. $2.4 + 5.8$ as $2 + 5$ and $0.4 + 0.8$ and combine the totals: $7 + 1.2 = 8.2$.



Counting on

Add two decimal numbers by adding the ones then the tenths/hundredths, e.g. $5.72 + 3.05$ as 5.72 add 3 (8.72) then add 0.05 (8.77).

Add near multiples of 1, e.g. $6.34 + 0.99$ or $5.63 + 0.9$.

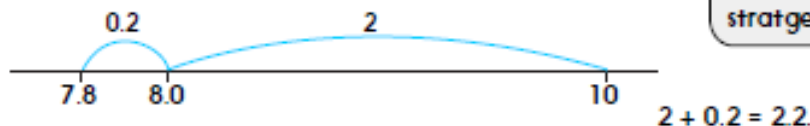


Count on from large numbers, e.g. $6834 + 3005$ as $9834 + 5$.

Using number facts

Number bonds to 1 and to the next whole number, e.g. $0.4 + 0.6$ or $5.7 + 0.3$.

Add to the next ten from a decimal number, e.g. $7.8 + 2.2 = 10$.



Subtracting by counting up is much less error prone.

Knowledge of number bonds underpins mental strategies.

- Subtraction

Taking away

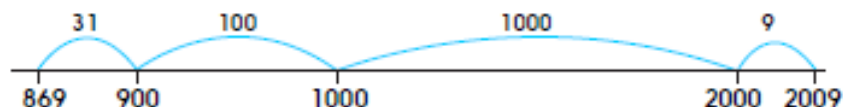
Using place value to subtract decimals, e.g. $4.58 - 0.08$ or $6.26 - 0.2$, etc. Take away multiples of powers of 10, e.g. $15,672 - 300$ or $4.82 - 2$ or $2.71 - 0.5$ or $4.68 - 0.02$.

Partition or count back, e.g. $3964 - 1051$ or $5.72 - 2.01$.

Subtract near multiples, e.g. $86,456 - 9999$ or $3.58 - 1.99$.

Counting up

Find a difference between two numbers by counting up from the smaller to the larger, e.g. $2009 - 869$.



$$1000 + 100 + 31 + 9 = 1140$$

Find change using shopkeepers' addition, e.g. buy toy for £6.89 using £10.



Using number facts

Derived facts from number bonds to 10 and 100, e.g. $2 - 0.45$ using $45 + 55 = 100$ or $3.00 - 0.86$ using $86 + 14 = 100$.

100	
86	14

Number bonds to £1, £10 and £100, e.g. $£4.00 - £3.86p = 14p$ or $£100 - £66$ using $66 + 34 = 100$.

Addition and subtraction are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using + and two using -) which can be written to express the relationship between 4 and 6 and 10. It is key to a good understanding of addition and subtraction that $6 + [] = 10$ and $10 - 6 = []$ are seen as ways of expressing the same question.

+ Written Addition

Expanded column addition for money leading to compact column addition for adding several amounts of money.

$$\begin{array}{r}
 \text{£}14 \quad 60\text{p} \quad 4\text{p} \\
 \text{£}28 \quad 70\text{p} \quad 8\text{p} \\
 + \text{£}12 \quad 20\text{p} \quad 6\text{p} \\
 \hline
 \text{£}1 \quad 10\text{p} \\
 \hline
 \text{£}55 \quad 60\text{p} \quad 8\text{p} \quad \text{£}55.68
 \end{array}$$

Expanded version first embeds understanding of place value.

Compact column addition to add pairs of 5-digit numbers.

Continue to use column addition to add towers of several larger numbers.

Use compact addition to add decimal numbers with up to two places.

$$\begin{array}{r}
 15.68 \\
 + 27.86 \\
 \hline
 11.1 \\
 \hline
 43.54
 \end{array}$$

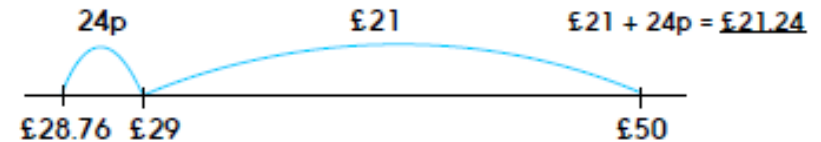
Adding fractions with related denominators, e.g. $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$.

- Written Subtraction

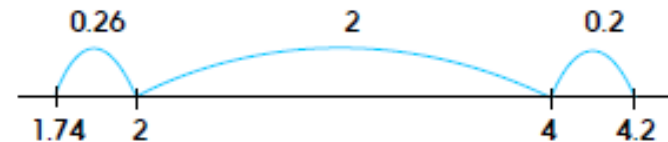
Compact column subtraction for numbers with up to 5 digits, e.g. $16,324 - 8516$.

$$\begin{array}{r}
 0 \quad 15 \quad 13 \quad 1 \quad 14 \\
 - \cancel{X} \quad \cancel{8} \quad \cancel{3} \quad \cancel{2} \quad \cancel{4} \\
 \hline
 8 \quad 5 \quad 1 \quad 6 \\
 \hline
 7 \quad 8 \quad 0 \quad 8
 \end{array}$$

Continue to use counting up subtraction for subtractions involving money, including finding change or, e.g. $\text{£}50 - \text{£}28.76$.



Use counting up subtraction to subtract decimal numbers, e.g. $4.2 - 1.74$.



$$2 + 0.26 + 0.2 = 2.46$$

Subtracting fractions with related denominators, e.g. $1\frac{1}{4} - \frac{3}{4}$ as $1\frac{1}{4} - \frac{3}{4}$ or $\frac{1}{4} - \frac{3}{4} = \frac{1}{4}$.

Equivalent fractions are the basis for + and - fractions.

Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $[\] \times 5 = 45$ and $45 \div 5 = [\]$ are seen as ways of expressing the same question.

\times Multiplication

Doubling and halving

Double amounts of money using partitioning, e.g. £6.73 doubled is double £6 (£12) plus double 73p (£1.46).

Use doubling and halving as a strategy in multiplying by 2, 4, 8, 5 and 20, e.g. $58 \times 5 = \frac{1}{2}$ of 58 (29) $\times 10$ (290).

$$\begin{array}{c} \text{£}6.73 \\ \swarrow \quad \searrow \\ \text{£}12 \quad + \quad \text{£}1.46 = \underline{\text{£}13.46} \end{array}$$

Partitioning remains a key skill throughout.

Grouping

Multiply decimals by 10, 100, 1000, e.g. $3.4 \times 100 = 340$.

100s	10s	1s	.	0.1s
		3	.	4
3	4	0		

Use partitioning to multiply friendly 2-digit and 3-digit numbers by single-digit numbers, e.g. 402×6 as 400×6 (2400) and 2×6 (12).

Use partitioning to multiply decimal numbers by single-digit numbers, e.g. 4.5×3 (4×3) + (4×0.5).

Multiply using near multiples by rounding, e.g. 32×29 as $(32 \times 30) - 32$.

Using number facts

Use times tables facts up to 12×12 to multiply multiples of the multiplier, e.g. $4 \times 6 = 24$ so $4 \times 6 = 240$ and $400 \times 6 = 2400$. Know square numbers and cube numbers.

Learning times tables involves BOTH multiplication and division facts.

\div Division

Doubling and halving

Halve amounts of money using partitioning, e.g. half of £14.84 as half of £14 and half of 84p.

$$\begin{array}{c} \text{£}14.84 \\ \swarrow \quad \searrow \\ \text{£}7 \quad + \quad 42\text{p} = \underline{\text{£}7.42} \end{array}$$

Use doubling and halving as a strategy in dividing by 2, 4, 8, 5 and 20, e.g. $115 \div 5$ as double 115 (230) $\div 10$.

Grouping

Divide numbers by 10, 100, 1000 to obtain decimal answers with up to three places, e.g. $340 \div 100 = 3.4$.

Use the 10th, 20th, 30th ... multiple of the divisor to divide friendly 2-digit and 3-digit numbers by single-digit numbers, e.g. $186 \div 6$ as 30×6 (180) and 1×6 (6).

Find unit and non-unit fractions of large amounts, e.g. $\frac{3}{5}$ of 265 is $3 \times (265 \div 5)$.

Using number facts

Use division facts from the times tables up to 12×12 to divide multiples of powers of ten of the divisor, e.g. $3600 \div 9$ using $36 \div 9$.

Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $[\] \times 5 = 45$ and $45 \div 5 = [\]$ are seen as ways of expressing the same question.

\times Written Multiplication

Short multiplication of 2-digit, 3-digit and 4-digit numbers by 1-digit numbers.

$$\begin{array}{r} 387 \\ \times 6 \\ \hline 54 \\ \hline 2322 \end{array}$$

Long multiplication of 2-digit, 3-digit and 4-digit numbers by teen numbers.

$$\begin{array}{r} 387 \\ \times 14 \\ \hline 3870 \\ 1548 \\ \hline 11 \\ \hline 5418 \end{array}$$

Grid multiplication of numbers with up to 2 decimal places by single-digit numbers.

£8.65 x 7

	£8	60p	5p	
x7	£56	£4.20	35p	£60.55

NB: Grid multiplication provides a default method for ALL children.

Multiplying fractions by single-digit numbers, e.g. $\frac{3}{4} \times 6 = \frac{18}{4}$ which is $4 \frac{3}{4} = 4 \frac{3}{4}$.

\div Written Division

Written version of a mental strategy for 3-digit \div 1-digit numbers.

$$\begin{array}{r} \square \times 6 = 326 \\ 50 \times 6 = 300 \\ \quad 26 \\ 4 \times 6 = 24 \\ \quad 2 \\ \hline 54 \text{ r } 2 \end{array}$$

Short division of 3-digit and 4-digit numbers by single-digit numbers.

$$\begin{array}{r} 1264 \\ 6 \overline{) 7584} \end{array}$$

The closer division is linked to multiplication the better.

Visual images are essential to multiplying and dividing fractions.

Addition and subtraction are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using + and two using -) which can be written to express the relationship between 4 and 6 and 10. It is key to a good understanding of addition and subtraction that $6 + [] = 10$ and $10 - 6 = []$ are seen as ways of expressing the same question.

+ Addition

Using place value

Count in 0.1s, 0.01s, 0.001s, e.g. knowing what 0.001 more than 6.725 is. Partitioning, e.g. $9.54 + 3.25$ as $9 + 3$ and $0.5 + 0.2$ and $0.04 + 0.05$ to get 12.79.

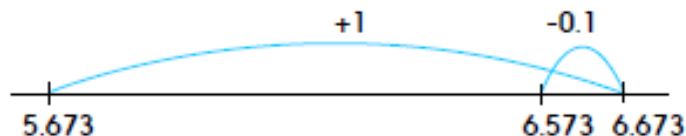
10s	1s	.	0.1s	$\frac{1}{10}$ s	0.01s	$\frac{1}{100}$ s
	9	.	5		4	
	3	.	2		5	
1	2	.	7		9	

Subtracting by counting up is much less error prone.

Counting on

Add two decimal numbers by adding the ones then the tenths/hundredths or thousandths, e.g. $6.314 + 3.006$ as 6.314 add 3 (9.314) then add 0.006 (9.32).

Add near multiples of 1, e.g. $6.345 + 0.999$ or $5.673 + 0.9$.



Count on from large numbers, e.g. $16,375 + 12,003$.

Using number facts

Number bonds to 1 and to the next multiple of 1, e.g. $0.63 + 0.37$ or $2.355 + 0.645$. Add to next ten, e.g. $4.62 + 0.38$.

5	
4.62	?

Knowledge of number bonds underpins mental strategies.

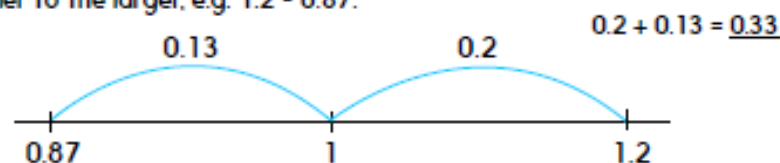
- Subtraction

Taking away

Use place value to subtract decimals, e.g. $7.782 - 0.08$ or $16.263 - 0.2$, etc. Take away multiples of powers of 10, e.g. $132,956 - 400$ or $686,109 - 40,000$ or $7.823 - 0.5$. Partition or count back, e.g. $3964 - 1051$ or $5.72 - 2.01$. Subtract near multiples, e.g. $360,078 - 99,998$ or $12.831 - 0.99$.

Counting up

Count up to subtract numbers from multiples of 10, 100, 1000, 10,000 Find a difference between two decimal numbers by counting up from the smaller to the larger, e.g. $1.2 - 0.87$.



Using number facts

Derived facts from number bonds to 10 and 100, e.g. $0.1 - 0.075$ using $75 + 25 = 100$ or $5 - 0.65$ using $65 + 35 = 100$.

Number bonds to £1, £10 and £100, e.g. $£7.00 - £4.37$ or $£100 - £66.20$ using $20p + 80p = £1$ and $£67 + £33 = £100$.

£100	
£67	£33

Addition and subtraction are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using + and two using -) which can be written to express the relationship between 4 and 6 and 10. It is key to a good understanding of addition and subtraction that $6 + [] = 10$ and $10 - 6 = []$ are seen as ways of expressing the same question.

+ Written Addition

Compact column addition for adding several large numbers and decimals with up to two places.

Compact column addition with money.

$\begin{array}{r} \text{£ } 14.64 \\ \text{£ } 28.78 \\ + \text{£ } 12.26 \\ \hline \text{£ } 55.68 \end{array}$	$\begin{array}{r} \text{£ } 14 \quad 60\text{p} \quad 4\text{p} \\ \text{£ } 28 \quad 70\text{p} \quad 8\text{p} \\ + \text{£ } 12 \quad 20\text{p} \quad 6\text{p} \\ \hline \text{£ } 55 \quad 60\text{p} \quad 8\text{p} \end{array}$	$\text{£ } 55.68$
--	--	-------------------

Adding fractions with unlike denominators.

e.g. $\frac{3}{4} + \frac{1}{3} = 1 \frac{1}{12}$ or $2 \frac{1}{4} + 1 \frac{1}{3} = 3 \frac{7}{12}$

$$\begin{aligned} \frac{3}{4} + \frac{1}{3} &= \frac{9}{12} + \frac{4}{12} \\ &= \frac{13}{12} \\ &= 1 \frac{1}{12} \end{aligned}$$

Children must be able to do expanded as well as compact to show understanding.

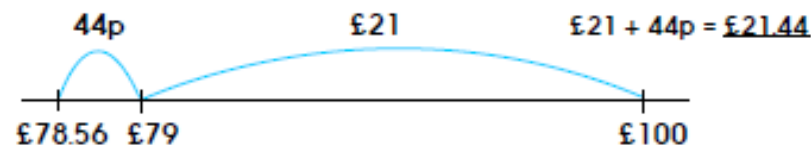
Understanding equivalent fractions is absolutely key here.

- Written Subtraction

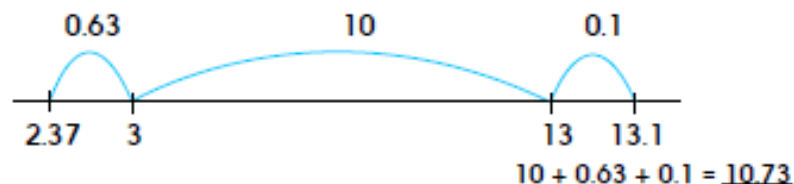
Compact column subtraction for large numbers.

$$\begin{array}{r} 2 \quad 14 \quad 7 \quad 15 \\ - 3 \quad 4 \quad 6 \quad 8 \quad 8 \\ \hline 1 \quad 6 \quad 4 \quad 5 \quad 8 \\ \hline 1 \quad 8 \quad 2 \quad 2 \quad 7 \end{array}$$

Use counting up subtraction when dealing with money, e.g. $\text{£ } 100 - \text{£ } 78.56$ or $\text{£ } 45.23 - \text{£ } 27.57$.



Use counting up subtraction to subtract decimal numbers, e.g. $13.1 - 2.37$.



Subtracting fractions with unlike denominators,

$$\begin{aligned} \text{e.g. } 1\frac{1}{4} - \frac{2}{3} &= \frac{5}{4} - \frac{2}{3} \\ &= \frac{15}{12} - \frac{8}{12} \\ &= \frac{7}{12} \end{aligned}$$

Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $\square \times 5 = 45$ and $45 \div 5 = \square$ are seen as ways of expressing the same question.

\times Multiplication

Doubling and halving

Double decimal numbers with up to 2-places using partitioning, e.g. 36.73 doubled is double 36 (72) plus double 0.73 (1.46).

$$\begin{array}{c} 36.73 \\ \swarrow \quad \searrow \\ 72 \quad + \quad 1.46 = \underline{73.46} \end{array}$$

Use doubling and halving as strategies in mental multiplication.

Grouping

Use partitioning as a strategy in mental multiplication, as appropriate, e.g. 3060×4 as $(3000 \times 4) + (60 \times 4)$ or 8.4×8 as 8×8 (64) and 0.4×8 (3.2)

Use factors in mental multiplication, e.g. 421×6 as 421×3 (1263) doubled (2526) or 3.42×5 as half of 3.42×10 .

Multiply decimal numbers using near multiples by rounding, e.g. 4.3×19 as 4.3×20 ($86 - 4.3$).

Using number facts

Use times tables facts up to 12×12 in mental multiplication of large numbers or numbers with up to two decimal places, e.g. $6 \times 4 = 24$ and $0.06 \times 4 = 0.24$.

Understanding how to partition numbers underpins many calculation strategies.

Division as grouping, i.e. the inverse of multiplication, is a key concept.

\div Division

Doubling and halving

Halve decimal numbers with up to 2-places using partitioning, e.g. half of 36.86 is half of 36 (18) plus half of 0.86 (0.43).

$$\begin{array}{c} 36.86 \\ \swarrow \quad \searrow \\ 18 \quad + \quad 0.43 = \underline{18.43} \end{array}$$

Use doubling and halving as strategies in mental division, e.g. $216 \div 4$ is half of 216 (108) and half of 108 (54).

Grouping

Use 10th, 20th, 30th, ... or 100th, 200th, 300th ... multiples of the divisor to divide large numbers, e.g. $378 \div 9$ as $40 \times 9 = 360$ and $2 \times 9 = 18$ so, the answer is 42.

Use test for divisibility, e.g. 135 divides by 3 as $1 + 3 + 5 = 9$ and 9 is in the 3x table.

$$\begin{array}{r} \square \times 9 = 378 \\ \underline{40 \times 9 = 360} \\ 18 \\ \underline{2 \times 9 = 18} \\ 0 \\ \underline{42} \end{array}$$

Using number facts

Use division facts from the times tables up to 12×12 to divide decimal numbers by single-digit numbers, e.g. $1.17 \div 3$ is $\frac{1}{100}$ of $117 \div 3$ (0.39).

Multiplication and division are inverse operations. Right from the start children should be taught these as related operations. There are four number sentences (two using \times and two using \div which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that $[\] \times 5 = 45$ and $45 \div 5 = [\]$ are seen as ways of expressing the same question.

\times Written Multiplication

Short multiplication of 2-digit, 3-digit and 4-digit numbers by 1-digit numbers.

$$\begin{array}{r} 3875 \\ \times 6 \\ \hline 543 \\ \hline 23250 \end{array}$$

Long multiplication of 2-digit, 3-digit and 4-digit numbers by 2-digit numbers.

$$\begin{array}{r} 258 \\ \times 16 \\ \hline 2580 \\ 1548 \\ \hline 4128 \end{array}$$

Short multiplication of decimal numbers using $\times 100$ and $\div 100$, e.g. 13.72×6 as $1372 \times 6 \div 100$.

Short multiplication of money, e.g. $\pounds 13.72 \times 6$ or $\pounds 23.67 \times 3$.

$$\begin{array}{r} \pounds 23.67 \\ \times 3 \\ \hline 122 \\ \hline \pounds 71.01 \end{array}$$

Grid multiplication of numbers with up to 2 decimal places by single-digit numbers.

Multiplying proper and improper fractions, e.g. $\frac{3}{4} \times \frac{2}{3}$.

\times	300	40	5	
20	6000	800	100	6900
6	1800	240	30	2070
				8970

Short versions of multiplication and division are more important and useful than the long versions.

\div Written Division

Short division of 3-digit and 4-digit numbers by single-digit numbers.

$$6 \overline{) 1264} \begin{array}{l} 21 \\ 10 \\ 26 \\ 44 \end{array}$$

Long division of 3-digit and 4-digit numbers by two-digit numbers.

$$\begin{array}{r} 200+50+1 \\ 15 \overline{) 3765} \\ \underline{3000} \\ 765 \\ \underline{750} \\ 15 \end{array} \begin{array}{l} 15 \\ 30 \\ 45 \\ 60 \\ 75 \\ 90 \end{array}$$

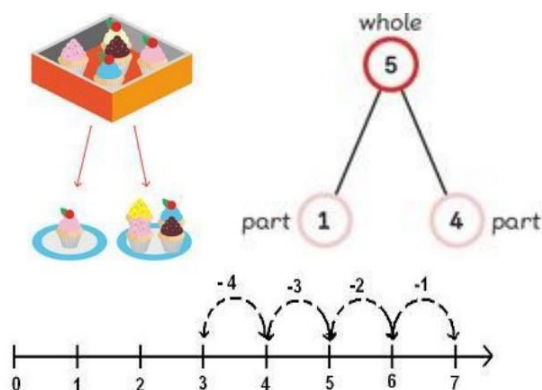
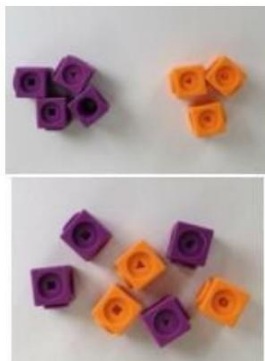
NB: Grid multiplication provides a default method for ALL children.

Divide fractions by whole numbers, e.g. $\frac{1}{4} \div 3 = \frac{1}{12}$.

CPA examples - Addition

Combining two parts to make a whole: part whole model.
Joining two groups and then recounting all objects (lots of practice making 10 and numbers to 10 e.g. $6 + 4 = 10$ or $3 + 5 = 8$)

$$3 + 4 = 7$$

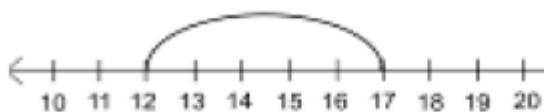


Starting at the bigger number and counting on



Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.

$$12 + 5 = 17$$

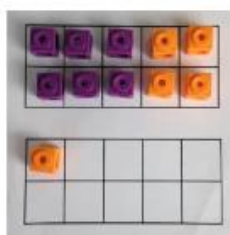


Start at the larger number on the number line and count on in ones or in one jump to find the answer.

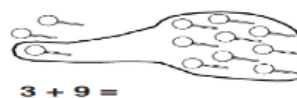
Regrouping to make 10



$$6 + 5 = 11$$

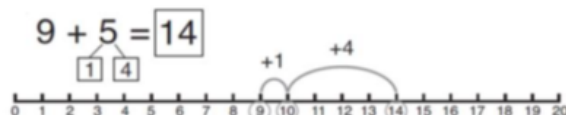


Start with the bigger number and use the smaller number to make 10.

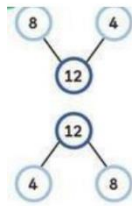
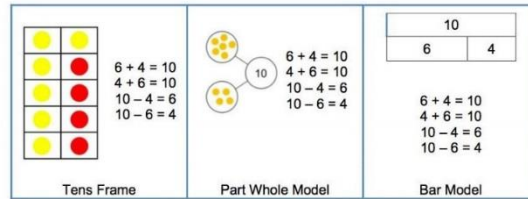


$$3 + 9 =$$

Use pictures or a number line. Regroup or partition the smaller number to make 10.



Number Bonds
Learn number bonds to 20 and demonstrate related facts. Addition and subtraction taught alongside each other as pupils need to see the relationship between the facts.



$8 + 4 = 12$
 $4 + 8 = 12$
 $12 - 8 = 4$
 $12 - 4 = 8$

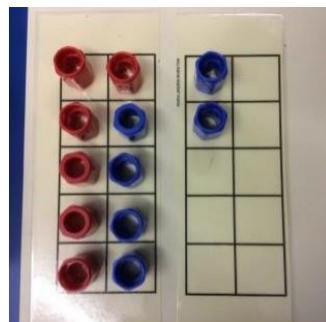
This is a family of addition and subtraction facts.

Add and subtract one digit numbers and two digit numbers to 20, including zero

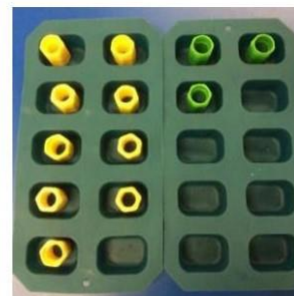
$8 + 1 = 9$



Bridging 10; $6 + 6 = 12$ Make 9 in one and 3 in the other. Take one from the 3 to make the 9 into a ten.... $10 + 2 = 12$ use ten frames, egg boxes and number lines to practice. Children should start with the larger number and add the smaller number



$6 + 6 = 12$



Make 9 in one and 3 in the other. Take one from the 3 to make the 9 into a ten.... $10 + 2 = 12$

Using concrete and pictorial representations to add a 2 digit number to a 1 digit number and a 2 digit number to a tens number.

Step 1 Add the ones.

tens	ones
2	5
+	3
<hr/>	
	8

Step 2 Add the tens.
1 ten + 2 tens = 3 tens

tens	ones
1	9
+	0
<hr/>	
2	9

$19 + 20 = 39$

Using concrete and pictorial representations to add two 2 digit numbers.

Step 1 Add the ones.
3 ones + 4 ones = 7 ones

tens	ones
2	3
+	4
<hr/>	
	7


Step 2 Add the tens.
2 tens + 1 ten = 3 tens

tens	ones
2	3
+	4
<hr/>	
3	7

$23 + 14 = 37$

Adding with renaming

Add 15 and 18.

Use  to help you add.



Step 1 Add the ones.
5 ones + 8 ones = 13 ones
Regroup the ones.
13 ones = 1 ten and 3 ones

tens	ones
1	5
+	8
<hr/>	
1	3

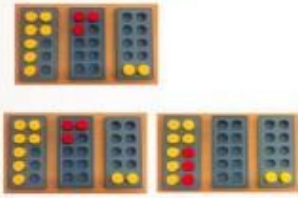
Step 2 Add the tens.
1 ten + 1 ten + 1 ten = 3 tens

tens	ones
1	5
+	8
<hr/>	
1	3
+	0
<hr/>	
3	3

$15 + 18 = 33$

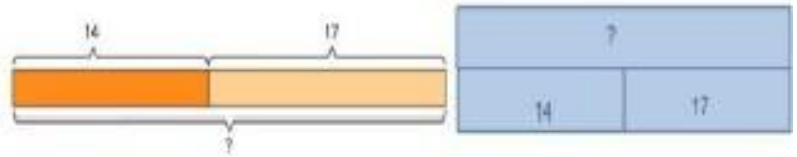
Using concrete and pictorial representations to add 3 single digit numbers.

$7+3+2 =$ leads to $10 + 2 =$



Using the bar model to find missing digits: It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.

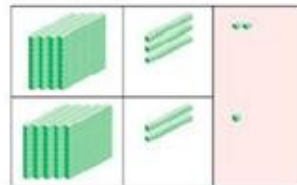
Helen has 14 breadsticks. Her friend has 17. How many do they have altogether?



Add two three digit numbers. Children need to first use equipment to support understanding of place value. Start without renaming then gradually move onto renaming.

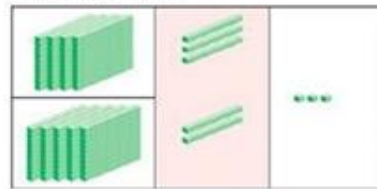
$432 + 521 =$

Step 1 Add the ones. 2 ones + 1 one = 3 ones



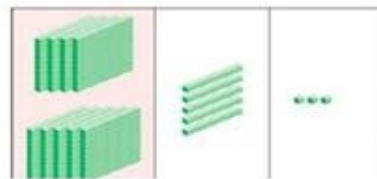
h	t	o
4	3	2
+	5	2
		1
		3

Step 2 Add the tens. 3 tens + 2 tens = 5 tens



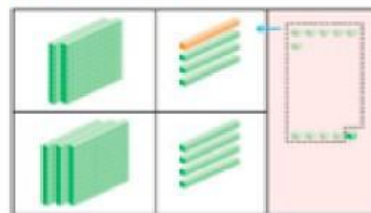
h	t	o
4	3	2
+	5	2
		1
		3
5	5	3

Step 3 Add the hundreds. 4 hundreds + 5 hundreds = 9 hundreds



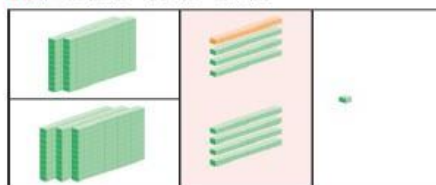
h	t	o
4	3	2
+	5	2
		1
		3
9	5	3

$236 + 345 =$



h	t	o
2	3	6
+	3	4
		5
		1

Step 2 Add the tens. 1 ten + 3 tens + 4 tens = 8 tens



h	t	o
2	3	6
+	3	4
		5
		1
		8

Step 3 Add the hundreds. 2 hundreds + 3 hundreds = 5 hundreds



h	t	o
2	3	6
+	3	4
		5
		1
		8
5	8	1

$236 + 345 = 581$

Bar Modelling

It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.

Bar Model to support understanding of problem solving:



A man sold 230 balloons at a carnival in the morning. He sold another 86 balloons in the evening. How many balloons did he sell in all?

?	
230	86
Morning	Afternoon

Adding numbers with up to 4 digits.

Again this should start with the children using equipment to support and lots of discussion about the values of digits.

$$\begin{array}{r} 2314 \\ + 4240 \\ \hline \end{array}$$



- Step 1 Add the ones.
4 ones + 0 ones = 4 ones
- Step 2 Add the tens.
1 tens + 4 tens = 5 tens
- Step 3 Add the hundreds.
3 hundreds + 2 hundreds = 5 hundreds
- Step 4 Add the thousands.
2 thousands + 4 thousands = 6 thousands

$$2314 + 4240 = 6554$$

Step 2 Add the tens. 7 tens + 3 tens + 1 ten = 11 tens.
Rename the tens. 11 tens = 1 hundred and 1 ten



$$\begin{array}{r} 5 \quad \overset{1}{0} \quad \overset{1}{7} \quad 8 \\ + 1 \quad 2 \quad 3 \quad 5 \\ \hline 1 \quad 3 \end{array}$$

Step 3 Add the hundreds.
6 hundreds + 2 hundreds + 1 hundred = 9 hundreds



$$\begin{array}{r} 5 \quad \overset{1}{0} \quad \overset{1}{7} \quad 8 \\ + 1 \quad 2 \quad 3 \quad 5 \\ \hline 9 \quad 1 \quad 3 \end{array}$$

Step 4 Add the thousands.
5 thousands + 1 thousand = 6 thousands



$$\begin{array}{r} 5 \quad \overset{1}{0} \quad \overset{1}{7} \quad 8 \\ + 1 \quad 2 \quad 3 \quad 5 \\ \hline 6 \quad 9 \quad 1 \quad 3 \end{array}$$

Using the bar model to find missing digits.

This is not a form of getting the correct answer but helping to guide children to the correct operation.

Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether?

?	
6860m	5470m

Adding numbers with more than 4 digits including decimals.

Using place value charts and place value counters is key when understanding adding decimals.

Using the bar model to find missing digits.

This is not a form of getting the correct answer but helping to guide children to the correct operation.

MacDonalds sold £9957.68 worth of hamburgers and £1238.5 worth of chicken nuggets. How much money did they take altogether?

?	
£957.68	£1238.5

Adding several numbers with up to 3 decimal places.

Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

CPA examples - Subtraction

Taking away should begin with physical objects: counters, cubes, Dienes etc

$6 - 3 = 3$

Subtraction by counting back

Let's Learn

Subtract by Counting Back

Subtract 3 from 15.

Count back 3 steps from 15.

$15 - 3 = 12$

There are 12 flowers left.

Subtracting a single digit number from a single digit number and a single digit from a two digit by crossing out pictures

Subtract by Crossing Out



$7 - 2 = 5$
5 ladybirds are left.

Subtracting using the part part whole model (include problem solving with missing dig-its). $7 - 5 = 2$



$7 - 5 = 2$
2 boats are not red.

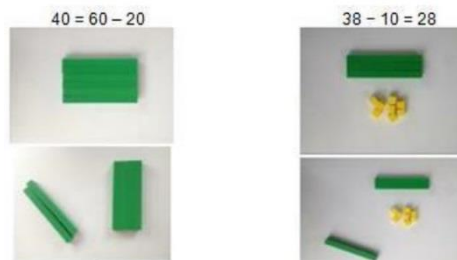
When subtracting using Dienes children should be taught to regroup (rename) a ten rod for 10 ones and then subtract from those



$20 - 4 = 16$

Subtracting Multiples of 10.

Using the vocabulary of 1 ten, two tens, etc, alongside 10, 20, 30 is important



Using concrete and pictorial representations to subtract a 1 digit number from a 2 digit number

Step 1 Subtract the ones.
 $8 \text{ ones} - 3 \text{ ones} = 5 \text{ ones}$



	tens	ones
-	2	8
		3
		5

Step 2 Subtract the tens.

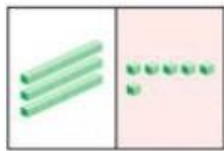


	tens	ones
-	2	8
		3
	2	5

$28 - 3 = 25$


Using concrete and pictorial representations to subtract a 2 digit number from a tens number

Step 1 Subtract the ones.



	tens	ones
	3	6
-	2	0
<hr/>		
		6

Step 2 Subtract the tens.
3 tens - 2 tens = 1 ten



	tens	ones
	3	6
-	2	0
<hr/>		
	1	6

36 - 20 = 16

Using concrete and pictorial representations to subtract a 2 digit number from a 2 digit number

Subtract 24 from 37.

Step 1 Subtract the ones.
7 ones - 4 ones = 3 ones

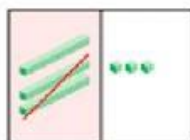


Use  to help you subtract.



	tens	ones
	3	7
-	2	4
<hr/>		
		3

Step 2 Subtract the tens.
3 tens - 2 tens = 1 ten



	tens	ones
	3	7
-	2	4
<hr/>		
	1	3

37 - 24 = 13

Recognise and use the inverse relationship between addition and subtraction.

?	
23	53

76	
23	?

Use this to check calculations and solve missing number problems.

Bar Modelling

It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.

315	
185	?

315 - 185 = ?

185 + ? = 315

?	
185	315

185 + 315 = ?

? - 185 = 315

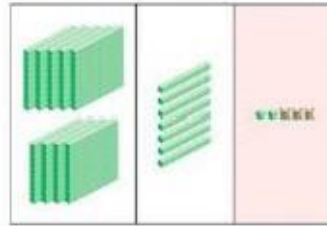
Subtract up to 3 digits from 3 digits.

Children need to first use equipment to support understanding of place value.

Only when children are secure with method should exchanging be introduced.

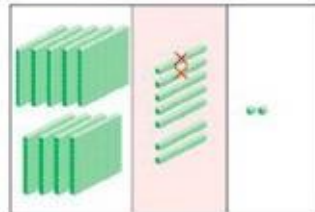
Subtract 723 from 975.

Step 1 Subtract the ones.
5 ones - 3 ones = 2 ones



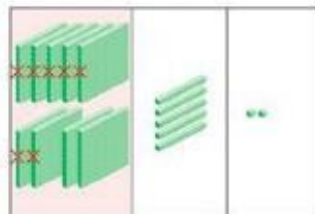
h	t	o
9	7	5
-	7	2
		2

Step 2 Subtract the tens.
7 tens - 2 tens = 5 tens



h	t	o
9	7	5
-	7	2
5		2

Step 3 Subtract the hundreds.
9 hundreds - 7 hundreds = 2 hundreds

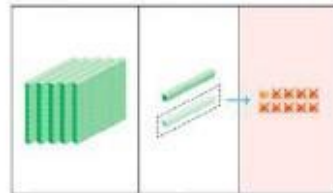
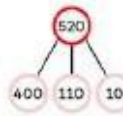


h	t	o
9	7	5
-	7	2
2	5	2

975 - 723 = 252

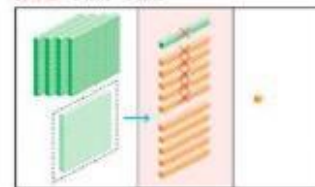
Subtract 269 from 520.

Step 1 Regroup 1 ten into 10 ones.
Subtract the ones.
10 ones - 9 ones = 1 one



h	t	o
5	2	0
-	2	6
		1

Step 2 Regroup 1 hundred into 10 tens.
Subtract the tens.
11 tens - 6 tens = 5 tens



h	t	o
5	2	0
-	2	6
5		1

Step 3 Subtract the hundreds.
4 hundreds - 2 hundreds = 2 hundreds

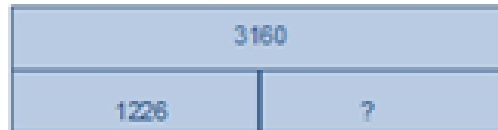


h	t	o
5	2	0
-	2	6
2	5	1

520 - 269 = 251

Using the bar model to find missing digits.

There are 3,160 books in a shop. 1,226 are in English and the rest are in French. How many French books are there?



Subtract with numbers up to four digits, including exchanging.

$$\begin{array}{r} 3437 \\ - 2016 \\ \hline 1421 \end{array}$$

Step 1 Subtract the ones.
7 ones - 6 ones = 1 one
 Step 2 Subtract the tens.
3 tens - 1 ten = 2 tens
 Step 3 Subtract the hundreds.
4 hundreds - 0 hundreds = 4 hundreds
 Step 4 Subtract the thousands.
3 thousands - 2 thousands = 1 thousand

CPA examples - Multiplication

Counting in Multiples of 2, 5 and 10 from zero.

Children should count the number of groups on their fingers as they are skip counting.



4 groups of 2 = 8

$4 \times 2 = 8$



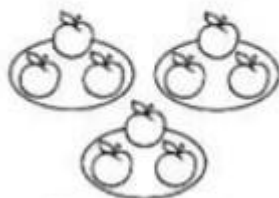
$2 \times 4 = 8$

When moving to pictorial/written calculations the language is important



This image represents two groups of 4 or 4 twice

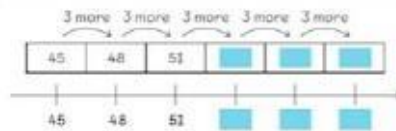
Solving Multiplication Problems using repeated addition



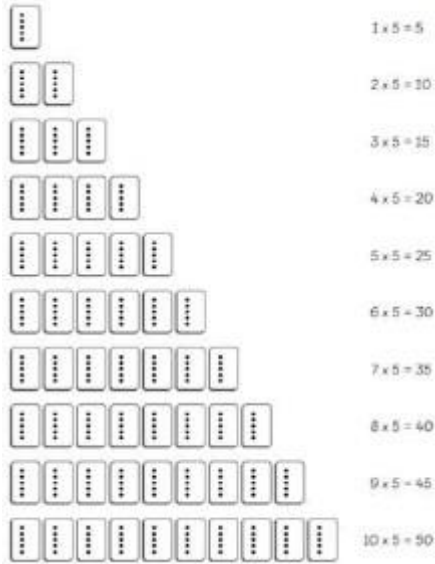
How many apples are there altogether?

$$3 + 3 + 3 = 9$$

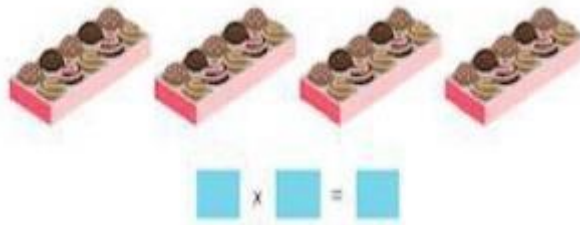
Skip counting in multiples of 2, 3, 5 and 10 from zero.



Recall and reuse multiplication facts for the 2, 5 and 10 times tables.



Use multiplication sign (X) and equals sign (=) when writing out multiplication tables.



Understand that multiplication is commutative

Pupils should understand that an array can represent different equations and that as multiplication is commutative the order doesn't affect the answer.

How many dots are there?

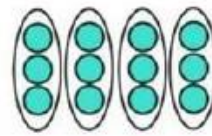


$2 \times 5 = 10$

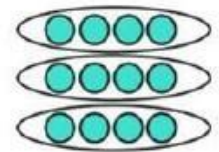


$5 \times 2 = 10$

2×5 is equal to 5×2 .



$12 = 3 \times 4$



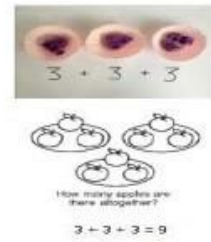
$12 = 4 \times 3$

Solve multiplication problems using arrays and repeated addition.



$$3 \times 5 = \square$$

$$5 \times 3 = \square$$

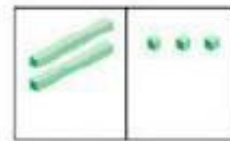


Children should be able to recall the 2, 5, 10, 3, 4 and 8 multiplication tables.

Multiply a 2 digit number by a 1 digit number.

Let's Learn

1 There are 4 groups of 23 fish. How do we multiply 23 by 4?



4 ones \times 3 = 12 ones
 12 ones = 1 ten 2 ones

Step 1 Multiply the ones by 4.

	t	o
	2	3
x		4
	1	2



2 tens \times 4 = 8 tens

Step 2 Multiply the tens by 4.

	t	o
	2	3
x		4
	8	0



12 + 80 = 92

Step 3 Add the products.

	t	o
	2	3
x		4
	1	2
+	8	0
	9	2

$$23 \times 4 = 92$$

There are 92 fish in 4 tanks.

Children know all times tables up to 12 x 12.
Children use expanded column multiplication

$$\begin{array}{r} 314 \\ \times 3 \\ \hline 12 \quad (3 \times 4) \\ 30 \quad (3 \times 10) \\ + 900 \quad (3 \times 300) \\ \hline 942 \end{array}$$



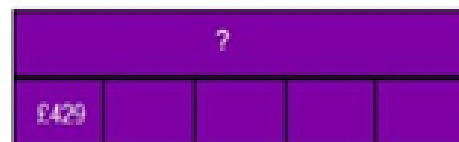
$$\begin{array}{r} 473 \\ \times 2 \\ \hline \end{array}$$

Multiply using the bar model

A computer costs 5 times as much as a television.
The television costs £42.

How much does the computer cost?

Cost of the computer

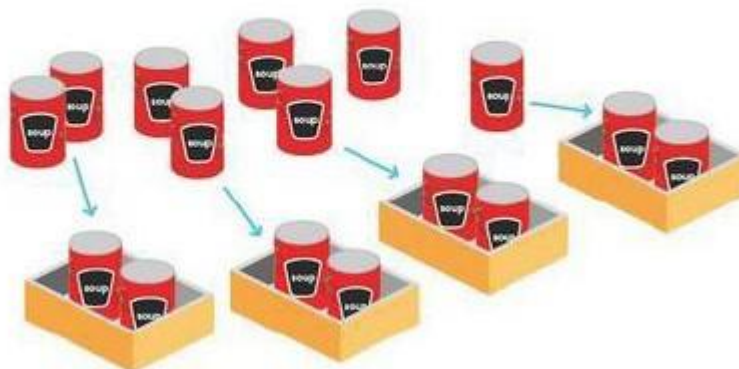


CPA examples - Division

Pupils should be taught to divide by working practically and the sharing should be shown below the whole to familiarise children with the concept of the whole.

$$10 \div 2 = 5$$

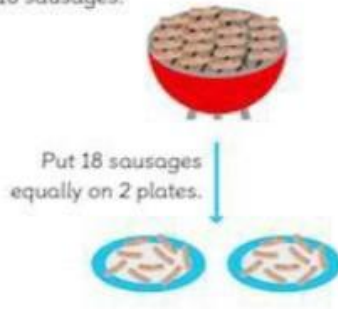
1 There are 8 cans.



There are 4 boxes of 2 cans.

Solve division problems in context by using concrete objects by sharing.

There are 18 sausages.



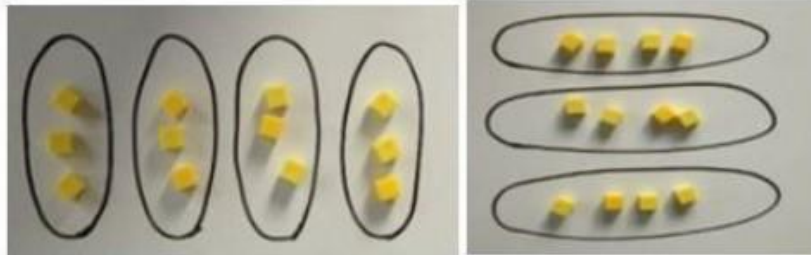
$2 \times 9 = 18$



There are 9 sausages on each plate.

$18 \div 2 = 9$

Solve division problems in context using arrays.



Solve division using grouping.

Put 10 buns in groups of 2.
How many plates are there?

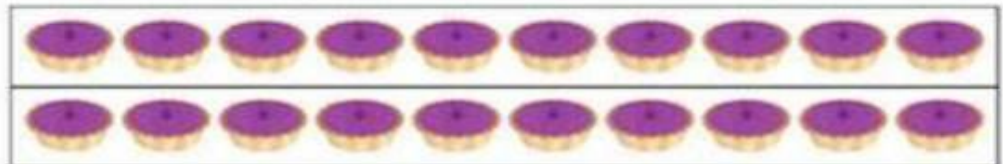


Put into groups of 5.

There are groups.

Use the inverse
This should be taught alongside both multiplication and division.

Make a family of multiplication and division facts.

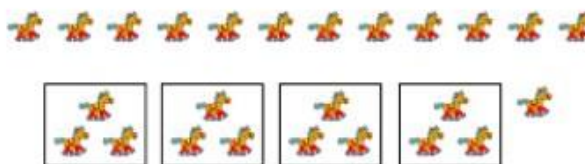


$2 \times 10 = 20$ ————— $20 \div 10 =$

$10 \times 2 = 20$ ————— $20 \div 2 =$

Dividing and grouping understanding the concept of remainders.

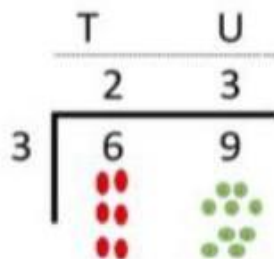
Start with using the real objects-or objects that represent the calculation.



$$13 \div 4 = 3 \text{ Remainder } 1$$

Dividing using short division

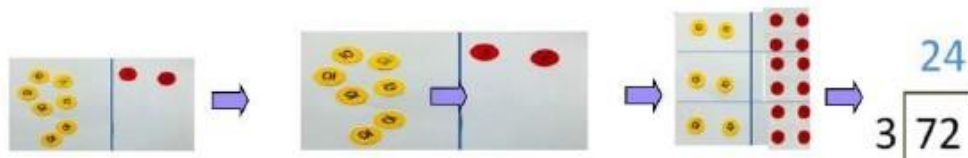
Once the children are secure with division as grouping and can demonstrate this on number lines, arrays etc. short division should be introduced for dividing larger 2 digit numbers. Initially with carefully chosen calculations requiring no remainders. Compare the layout of short division and that of an array.



Remind children of correct place value, that 69 is equal to 60 and 9, but in short division, pose:

- How many 3's in 6? = 2, and record it above the 6 tens.
- How many 3's in 9? = 3, and record it above the 9 ones.

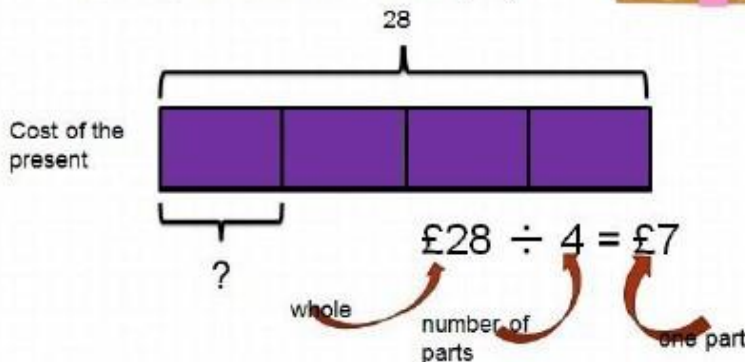
Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. $72 \div 3$), and be taught to 'carry' the remainder onto the next digit.



Bar Modelling

It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.

Four children bought a present for £28. They shared the costs equally. How much did each child pay?



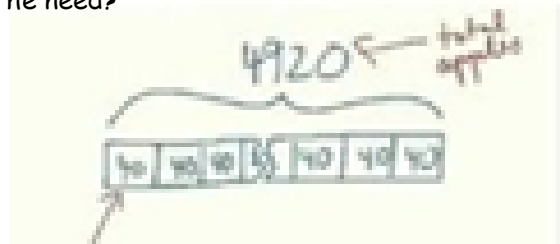
Dividing 3 digit numbers by a 1 digit number using short division.

	H	T	U	
	0	2	5	r1
5	1	2	6	



Using the bar model to support division

Frank has 4920 apples. He needs to put them into baskets of 40. How many baskets does he need?



Long division to divide by 2 digit number.

Try this equation; $848 \div 16$

Approximation $800 \div 16 = 50$

$\begin{array}{r} 053 \\ 16 \overline{) 848} \\ \underline{-80} \\ 48 \\ \underline{-48} \\ 0 \end{array}$	Start with the largest place holder in the case it will be the hundreds column. 8 - 16 not possible. So put a 0 above the hundreds column. Carry the 84 over to the Tens column! 84 - 16 = 16 x 5 = 80 84 - 80 = 4
--	---

$48 - 16 = 3$
No remainder

Division

$564 \div 13$

$\begin{array}{r} 43.38 \\ 13 \overline{) 564.00} \\ \underline{52} \\ 44 \\ \underline{39} \\ 50 \\ \underline{39} \\ 110 \\ \underline{117} \\ 30 \end{array}$	Using known multiplication facts <table border="1" style="width: 100%; text-align: center;"> <tr><td>1</td><td>13</td></tr> <tr><td>2</td><td>26</td></tr> <tr><td>4</td><td>52</td></tr> <tr><td>5</td><td>65</td></tr> <tr><td>8</td><td>104</td></tr> <tr><td>10</td><td>130</td></tr> <tr><td>20</td><td>260</td></tr> </table>	1	13	2	26	4	52	5	65	8	104	10	130	20	260
1	13														
2	26														
4	52														
5	65														
8	104														
10	130														
20	260														

$564 \div 13 = 43 \text{ r } 5 = 43 \frac{5}{13} = 43.4 \text{ (to 1dp)}$