



Calculation Policy

Ratified by governing body: Autumn 17

Date for review: Autumn 2020

A handwritten signature in black ink, appearing to read 'Anne Marie Renshaw', is positioned above the printed name.

Reverend Anne Marie Renshaw

Chair of Governors

Introduction

This policy has been written by the White Rose Maths Hub. It follows the CPA approach (concrete, pictorial, abstract) which develops the skills so that children master mathematical concepts in line with the National Curriculum. It also shows visual representations of how to achieve concepts. It is every child's right to acquire sound numeracy skills to enable them to function confidently within society.

Aims

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding
- To ensure that children can explain their methods and reasoning using well developed mathematical vocabulary
- To ensure that children can apply written calculation methods to a range of problem solving and mathematical investigations
- To ensure calculation skills are underpinned by sound estimation and reasoning about number skills

What are children learning?

Year group expectations are available on our website (www.messingprimaryschool.co.uk).

In Early Years, children are learning about number conservation and 1 to 1 matching through play based experiences. They learn to match concrete sets with abstract symbols (numerals). They are introduced to addition, subtraction and the + - = signs. Children learn to find 1 more and 1 less to at least 20.

In KS1, pupils use number lines alongside concrete apparatus to support calculations. They add and subtract using part/whole models, counting forward and backward.

In KS2 pupils move to formal, efficient written calculations.

How do we support their learning?

- Use of visual models in different representations to support calculations
- Development of language so children can explain their understanding and methods
- Weekly skills checks using Assertive Mentoring and basing medium term plans on gaps from these and half termly assessments
- Daily times tables practice in KS2 and regular times table practice alongside specific teaching of times tables and related division facts
- Chanting, counting forwards and backwards in different steps.
- Online games and activities to support learning (e.g. Activelearn/Abacus)

- Relating learning to real life experiences. Learning inside and outside the classroom
- Giving opportunities to apply calculations to a range of problems

How are individual needs supported and developed? (See below)

Special Educational Needs

Children with Special Educational Needs, whether they have a specific difficulty or a particular talent in Mathematics have access to the same broadly balanced curriculum as their peers. It is the teacher's role to adapt tasks and activities to the individual needs of each child in his/her class. This should be clearly outlined in the teacher's planning.

Equal opportunities.

Within the teaching of mathematics we aim to ensure that all pupils regardless of age, disability, race, religion or belief, sex, attainment and background, have full access to the mathematics curriculum. Teachers respond to diverse learning needs so that pupils are appropriately supported and challenged to experience success in learning and achieve as high a standard as possible.

As an educationally inclusive school the teaching and learning, achievements, attitudes and well-being of every young person matters. We take into account pupils' varied life experiences and needs, providing equal opportunities for all pupils, whatever their age, disability, race, religion or belief, sex, attainment and background, to ensure that every child really does matter.

Roles and responsibilities

Teachers are responsible for providing a range of accessible visual models in lessons and using questioning to develop concepts. Teachers' weekly plans and assessments are available for monitoring on a weekly basis. Evaluation takes place during staff discussions and Headteacher/Maths subject lead monitoring. Books are sampled on a regular basis, lessons are observed and feedback given, pupils share their work. The Headteacher has a focus for monitoring linked to the School Development Plan, and this is fed back to and discussed at governor's meetings on a termly basis. Pupil progress meetings are held with class teachers on a half termly basis to discuss children's progress and next steps for planning and teaching.

How do we monitor progress and achievement and report findings?

Effective assessment (See assessment policy) is essential to quality teaching and learning. Assessment for Learning (AfL) is a tool utilised by the school to raise attainment and accelerate progress. Good assessment practice ensures lesson planning is relevant and is based on a sound knowledge of the pupils' learning styles, attainment, progress and the next steps in their learning.

Regular feedback is given to pupils (see the schools' Marking Policy) and helps them to understand how to be successful, what they have achieved and what they need to do to improve further.

The pupils at Messing Primary School take summative assessments in line with statutory requirements. Reception children are assessed against the Early Learning Goals. Children take part in maths SATs in Year 2 (which supports the teachers' overall assessment of their attainment) and Year 6.

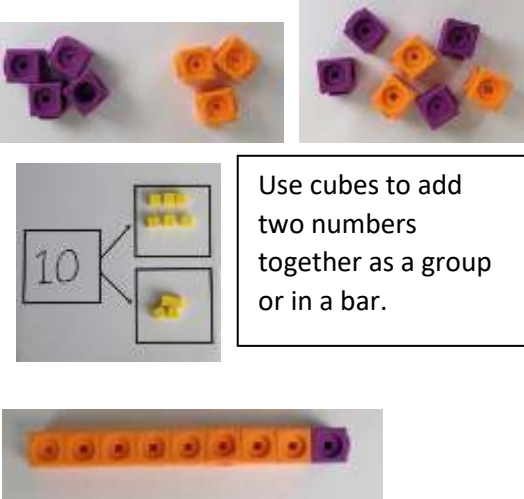
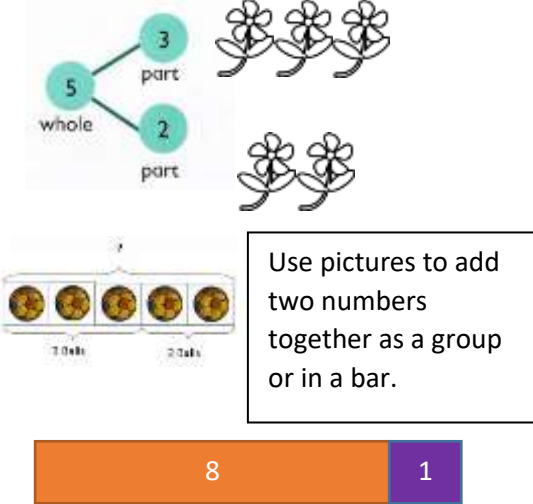
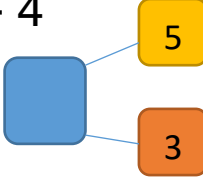

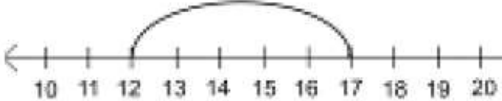
Weekly skills checks using Assertive Mentoring monitors children attainment and half termly tests are analysed and inform medium term and short term planning.

When do we review?

We work with other schools in the Consortium to share good practice and develop new initiatives. Needs of staff are reviewed through performance management and outcomes from monitoring. Staff attend training when necessary and practice is disseminated through the school through staff meetings and informal coaching sessions.

Progression in Calculations

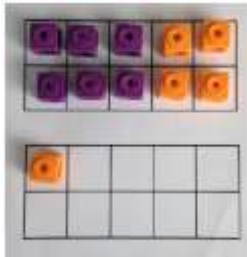
Addition

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model</p>	 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>

Regrouping to make 10.



$$6 + 5 = 11$$

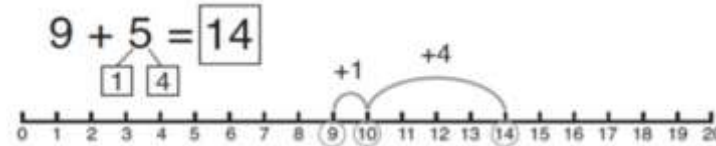


Start with the bigger number and use the smaller number to make 10.



$$3 + 9 =$$

Use pictures or a number line. Regroup or partition the smaller number to make 10.



$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10. How many more do I add on now?

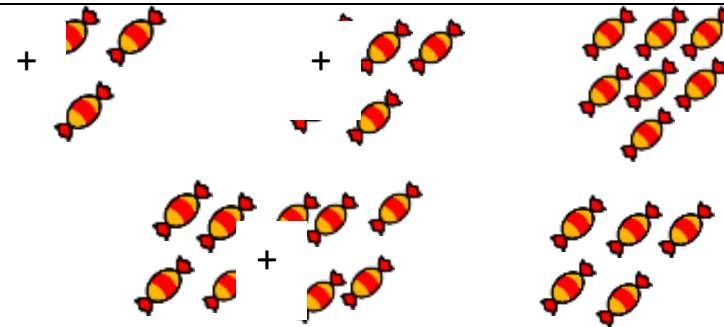
Adding three single digits

$$4 + 7 + 6 = 17$$

Put 4 and 6 together to make 10. Add on 7.



Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.



Add together three groups of objects. Draw a picture to recombine the groups to make 10.

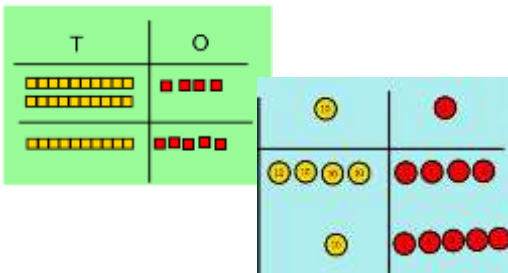
$$\begin{aligned} (4 + 7) + 6 &= 10 + 7 \\ &= 17 \end{aligned}$$

Combine the two numbers that make 10 and then add on the remainder.

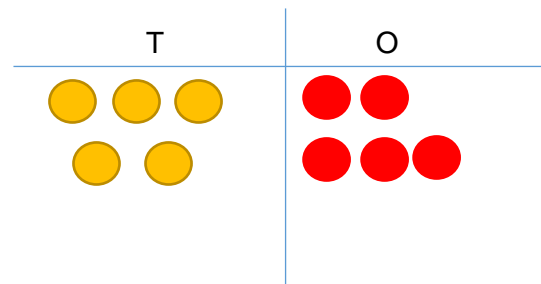
Column method- no regrouping

$$24 + 15 =$$

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.



After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.



Calculations

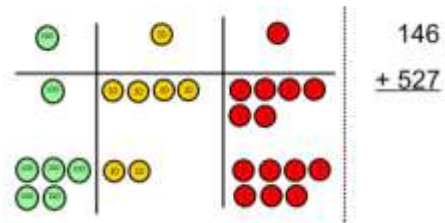
$$21 + 42 =$$

21

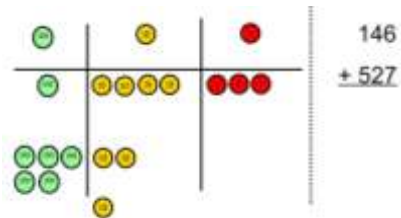
+ 42

Column method- regrouping

Make both numbers on a place value grid.



Add up the units and exchange 10 ones for one 10.

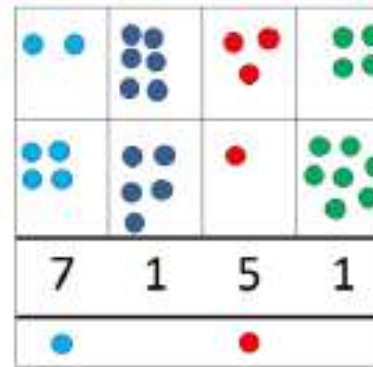


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

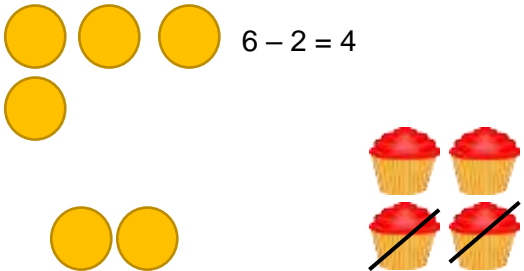
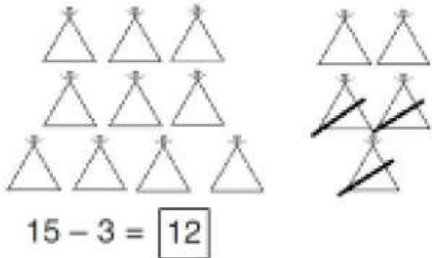


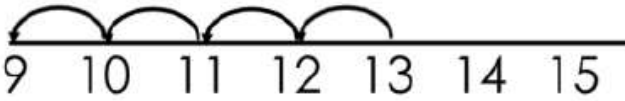
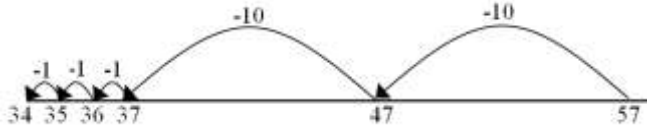
As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

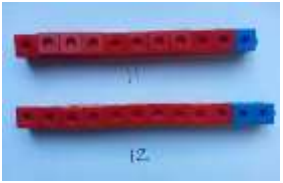
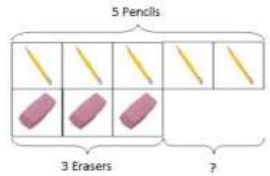
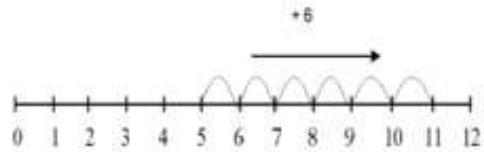
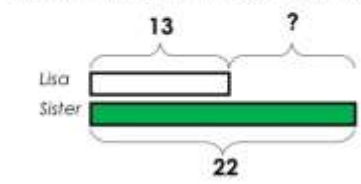
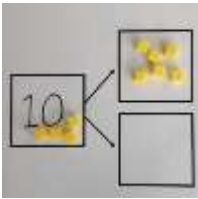
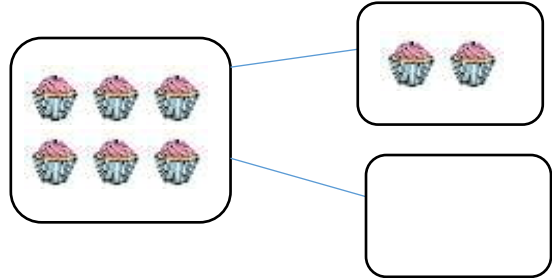
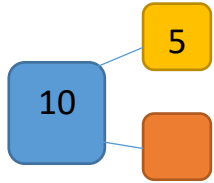

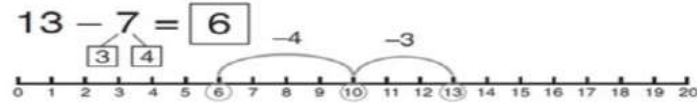
$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \end{array}$$

$$\begin{array}{r} £ 23.59 \\ + £ 7.55 \\ \hline £ 31.14 \end{array}$$

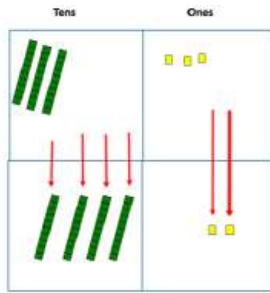
$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ - 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 2 = 4$</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p>	<p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p>
<p>Counting back</p>	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p>  <p>$13 - 4$</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p> 	<p>Count back on a number line or number track</p>  <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p>  <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p>

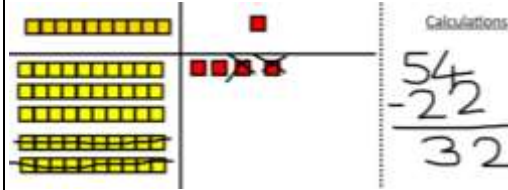
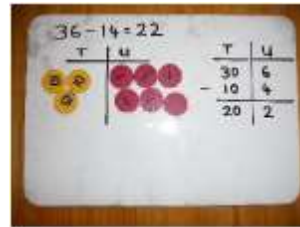
<h3>Find the difference</h3>	<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p>  <p>Use basic bar models with items to find the difference</p>	 <p>Count on to find the difference.</p> <p>Comparison Bar Models</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p> 	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
<h3>Part Part Whole Model</h3>	<p>Link to addition- use the part whole model to help explain the inverse between addition and subtraction.</p>  <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p>$10 - 6 =$</p>	<p>Use a pictorial representation of objects to show the part part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>
<h3>Make 10</h3>	<p>$14 - 9 =$</p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</p>	<p>$13 - 7 = 6$</p>  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p>$16 - 8 =$</p> <p>How many do we take off to reach the next 10?</p> <p>How many do we have left to take off?</p>

Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to help to show working.

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

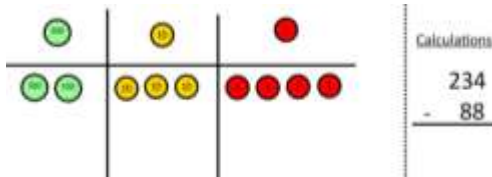
This will lead to a clear written column subtraction.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

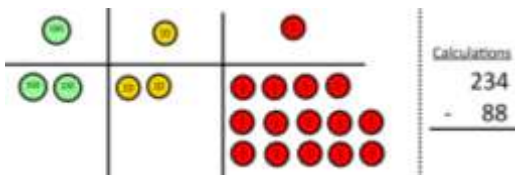
Make the larger number with the place value counters



Calculations

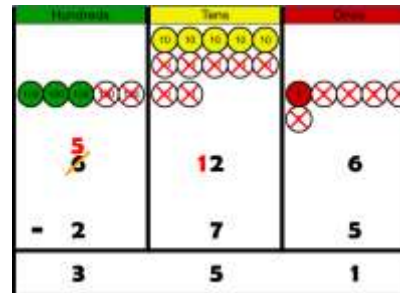
$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and

knows when to exchange/regroup.

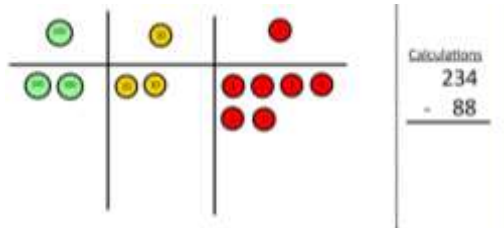


Children can start their formal written method by partitioning the number into clear place value columns.

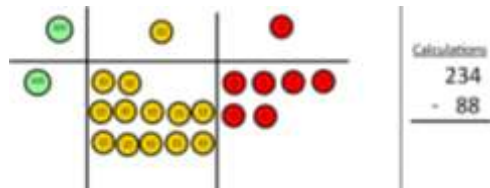


Moving forward the children use a more compact method.

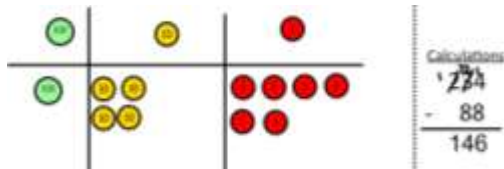
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction

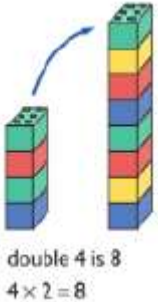

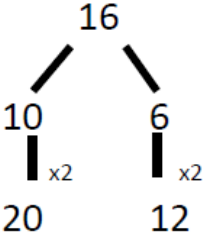
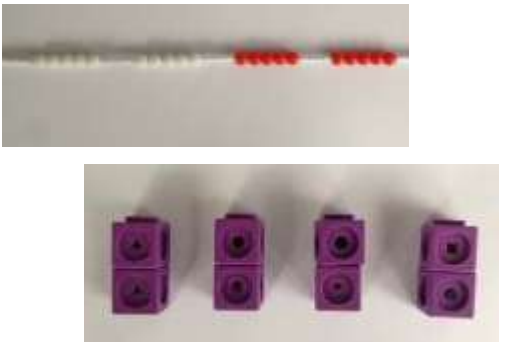
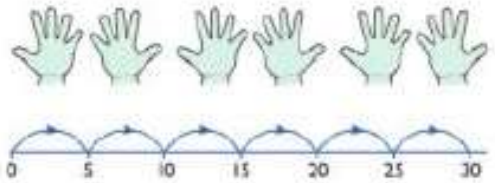


Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.




This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad \cancel{0} \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
<p>Counting in multiples</p>	 <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>


Repeated addition

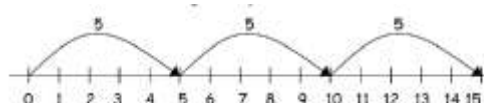
Use different objects to add equal groups.

$$3 + 3 + 3$$

There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



2 add 2 add 2 equals 6



$$5 + 5 + 5 = 15$$

Write addition sentences to describe objects and pictures.




$$2 + 2 + 2 + 2 + 2 = 10$$


Arrays- showing commutative multiplication


Create arrays using counters/ cubes to show multiplication sentences.

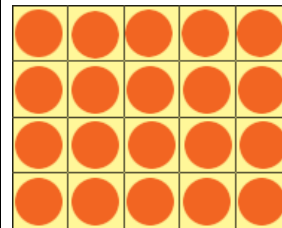


Draw arrays in different rotations to find **commutative** multiplication sentences.



$$4 \times 2 = 8$$


$$2 \times 4 = 8$$


$$4 \times 2 = 8$$


Link arrays to area of rectangles.

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Grid Method

Show the link with arrays to first introduce the grid method.

x	10	3
4		

4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

x	T	U

4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.

	100	10	1

Calculations
4 x 126

Fill each row with 126.

	100	10	1
	100	10	1
	100	10	1
	100	10	1

Calculations
4 x 126

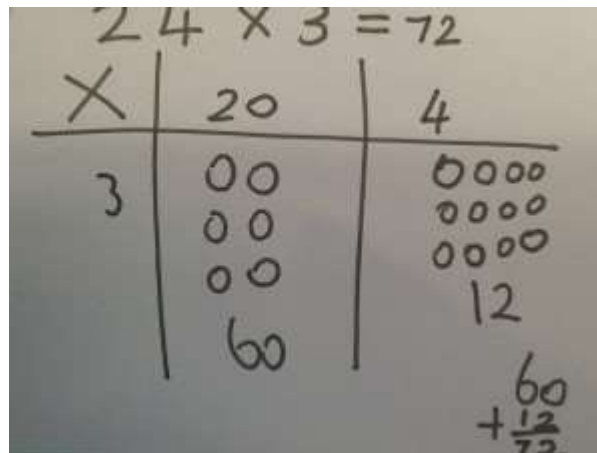
Add up each column, starting with the ones making any exchanges needed.

	100	10	1
	100	10	1
	100	10	1
	100	10	1

Then you have your answer.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	30	5
7	210	35

$$210 + 35 = 245$$

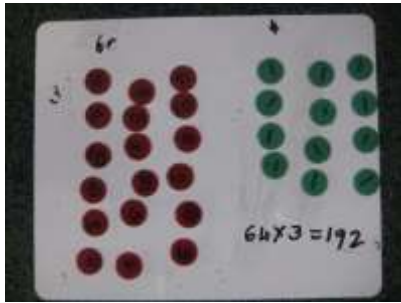
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

x	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

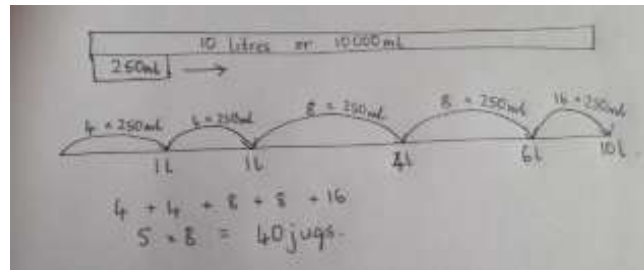
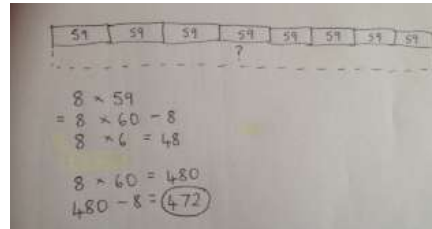
Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

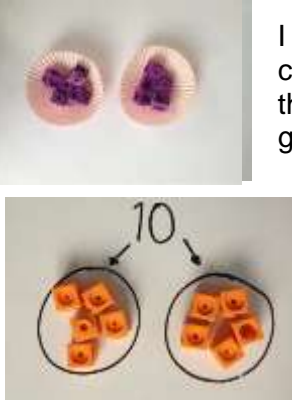
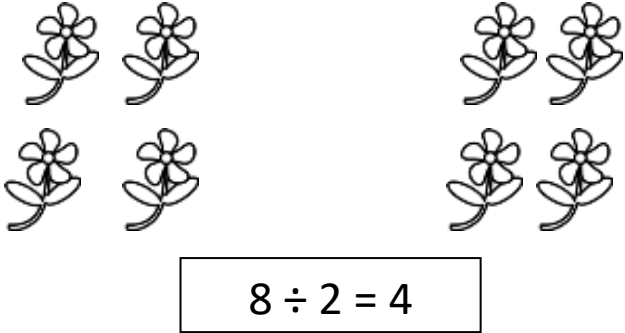
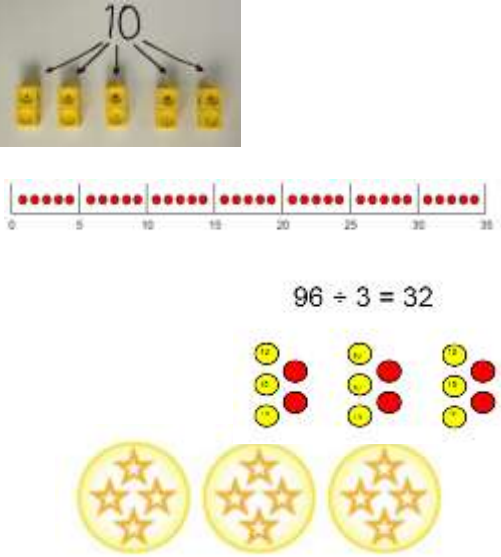
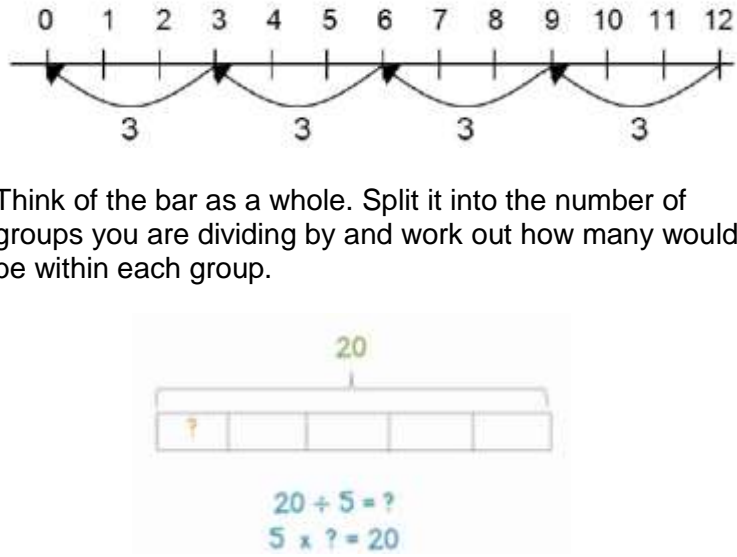
$$\begin{array}{r}
 32 \\
 \times 24 \\
 \hline
 8 \quad (4 \times 2) \\
 120 \quad (4 \times 30) \\
 40 \quad (20 \times 2) \\
 600 \quad (20 \times 30) \\
 \hline
 768
 \end{array}$$

$$\begin{array}{r}
 7 4 \\
 \times 6 3 \\
 \hline
 1 2 \\
 2 1 0 \\
 2 4 0 \\
 + 1 2 0 0 \\
 \hline
 4 6 6 2
 \end{array}$$

This moves to the more compact method.

$$\begin{array}{r}
 2 3 1 \\
 1342 \\
 \times 18 \\
 \hline
 13420 \\
 10736 \\
 \hline
 24156 \\
 1
 \end{array}$$

Division

Objective and Strategies	Concrete	Pictorial	Abstract
<p>Sharing objects into groups</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p> 	<p>Share 9 buns between three people.</p> $9 \div 3 = 3$
<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  $96 \div 3 = 32$	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p> $20 \div 5 = ?$ $5 \times ? = 20$	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>

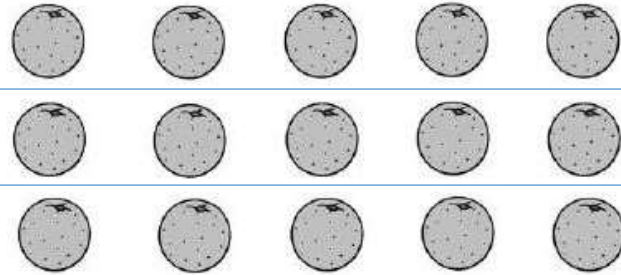
Division within arrays



Link division to multiplication by creating an array and thinking about the

number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



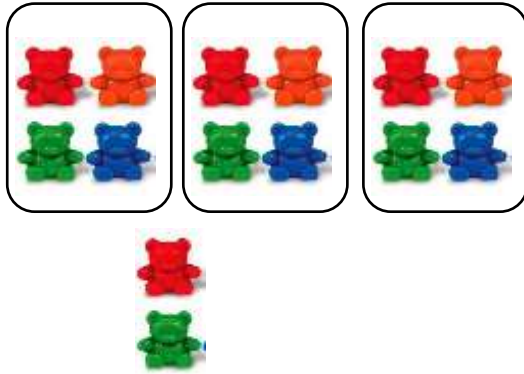
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

$7 \times 4 = 28$
 $4 \times 7 = 28$
 $28 \div 7 = 4$
 $28 \div 4 = 7$

Division with a remainder

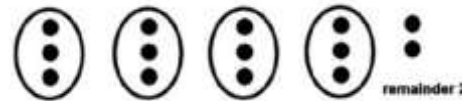
$14 \div 3 =$
 Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



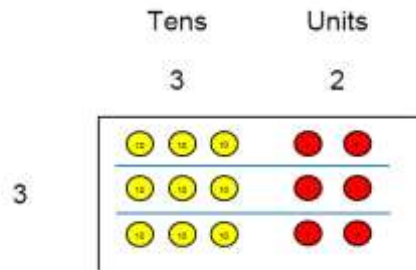
Complete written divisions and show the remainder using r.

$$29 \div 8 = 3 \text{ REMAINDER } 5$$

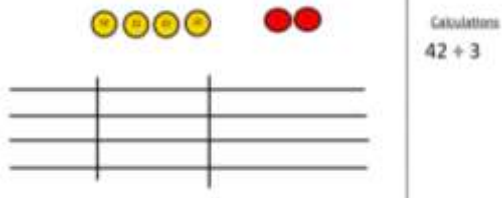
↑
↑
↑
↑

 dividend divisor quotient remainder

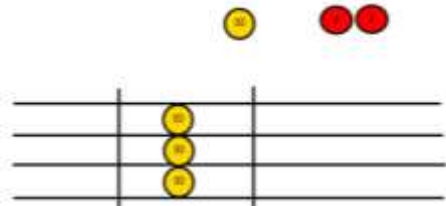
Short division



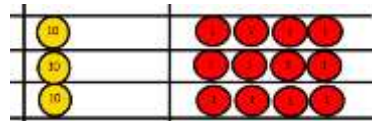
Use place value counters to divide using the bus stop method alongside



$42 \div 3 =$
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

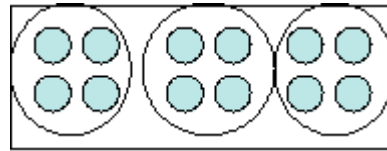


We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 4272} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$$